Additional Mathematics Trigonometry Exercise 1:

1. **Prove** $(\sec x - \cos x)(1 + \cot x + \tan x) = \dfrac{\sec^2 x}{\csc x} + \dfrac{\sec x}{\csc^2 x}$.

2. **Prove** $\dfrac{[\sec(-x) + \sin(-x - 90^\circ)]}{[\csc(540^\circ - x) - \cos(270^\circ + x)]} = \tan^3 x$

3. **If** $\sec x(\sec x - \tan x) = x$, **prove** $\sin x = \dfrac{1 - x}{x}$.

4. **If** $\sin^2 x + \sin x - 1 = 0$, **Without solving the values of x, prove that:**
   a. $\cos^4 x + \cos^2 x - 1 = 0$
   b. $\cos^8 x + \cos^6 x + \cos^2 x - 1 = 0$

5. **Prove** $\dfrac{(\cos^2 x - \sin x \cos x + \tan x)}{(\cos^2 x + \sin x \cos x - \tan x)} = (1 + \tan^3 x)(1 - \tan^3 x)$.

6. **If** $\sin x$ **and** $\csc x$ **are the two roots of the equation** $3x^2 + hx - 1 = 0$, **where** $\dfrac{\pi}{2} < x < \pi$, **prove** $h = -\dfrac{7\sqrt{10}}{10}$.

7. **If** $\sin x$ **and** $\cos x$ **are two roots of the equation** $kx^2 - 4x + 3 = 0$, **find the possible value(s) of k.**

8. **Given that** $\dfrac{\sin^2 A}{1 + 2\cos^2 A} = \dfrac{3}{19}$, **where** $90^\circ < A < 180^\circ$. **Prove** $\dfrac{\sin A}{1 + 2\cos A} = -1$.

Useful Identities and Hints:

1. $\sin^2 \theta + \cos^2 \vartheta = 1$
2. $sex^2\theta - 1 = \tan^2 \theta$
3. $\csc^2 \theta - 1 = \cot^2 \theta$
4. 單數轉 **Functions,** 正負睇 **Quadrants.**
5. $\sec\theta = \dfrac{1}{\cos\theta}$, $\csc\theta = \dfrac{1}{\sin\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$
6. **Sum of roots =** $-\dfrac{b}{a}$, **Product of roots =** $\dfrac{c}{a}$.

The End.