

Mathematics

數學科

Deductive Geometry

演繹幾何

Bottom-up Approach!

(由下至上法)!

Association Approach!

(聯想法)!



Contents 目錄:

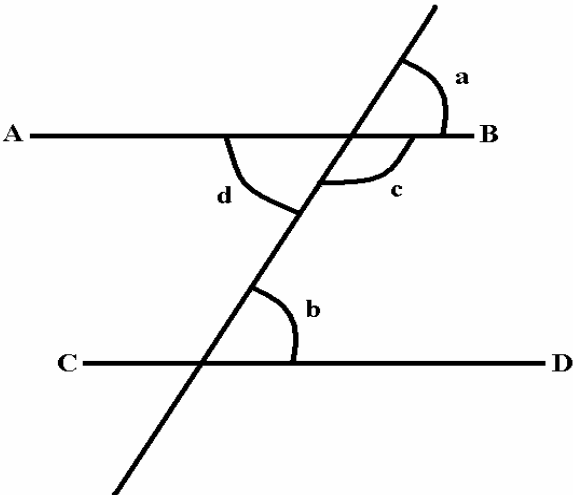
Deductive Geometry 演繹幾何

- **Summary of Geometry Proving Skills** 幾何學證明技巧總結
- **Method for solving Geometry Questions** 解幾何問題方法
- **Construction of Geometry figures** 幾何繪圖

(A) Summary of Geometry Proving Skills 幾何學證明技巧總結

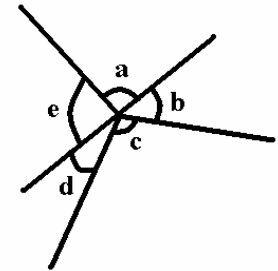
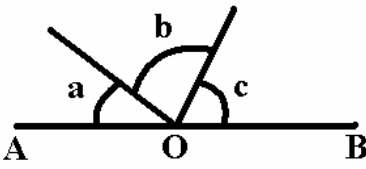
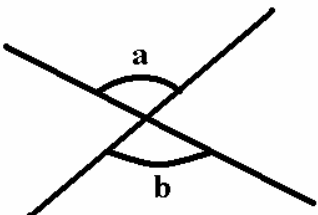
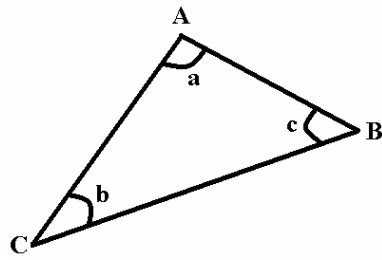
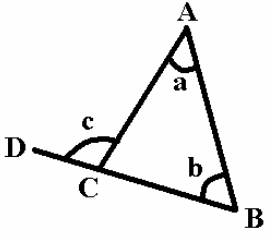
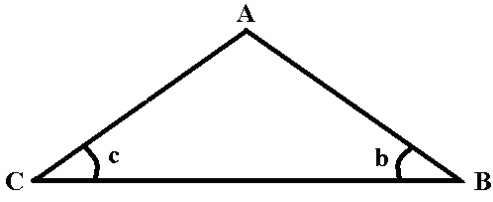
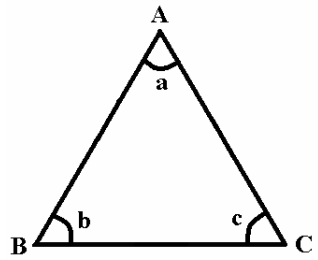
1. Relationship between parallel lines and its angles.

平行線及其角之關係.

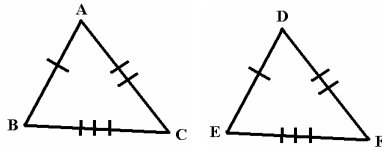
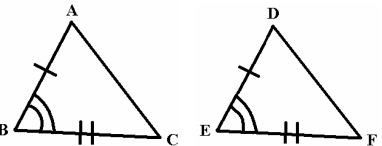
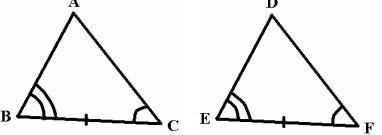
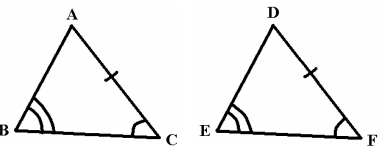
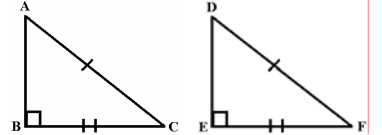
<p>i. If $\angle a = \angle b$, then $AB \parallel CD$. (Corr. \angles equal.) 如果 $\angle a = \angle b$, 則 $AB \parallel CD$. (對應角相等.)</p> <p>ii. If $\angle b = \angle d$, then $AB \parallel CD$. (Alt. \angles equal.) 如果 $\angle b = \angle d$, 則 $AB \parallel CD$. (錯角相等.)</p> <p>iii. If $\angle b + \angle c = 180^\circ$, then $AB \parallel CD$. (Int. \angles supplementary) 如果 $\angle b + \angle c = 180^\circ$, 則 $AB \parallel CD$. (同旁內角互補.)</p>	
<p>iv. If $AB \parallel CD$, then $\angle a = \angle b$. (Corr. \angles, $AB \parallel CD$) 如果 $AB \parallel CD$, 則 $\angle a = \angle b$. (對應角, $AB \parallel CD$)</p> <p>v. If $AB \parallel CD$, then $\angle b = \angle d$. (Alt. \angles, $AB \parallel CD$) 如果 $AB \parallel CD$, 則 $\angle b = \angle d$. (錯角, $AB \parallel CD$)</p> <p>vi. If $AB \parallel CD$, then $\angle b + \angle c = 180^\circ$. (Int. \angles, $AB \parallel CD$) If $AB \parallel CD$, then $\angle b + \angle c = 180^\circ$. (同旁內角, $AB \parallel CD$)</p>	

2. Properties of angles at a point, on a straight line and in a triangle.

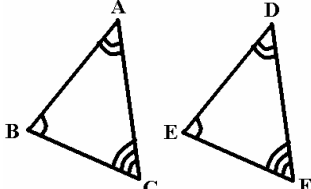
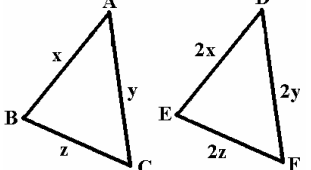
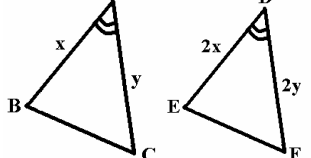
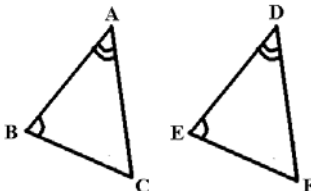
角在一點上, 一直線上和一三角形內之特性.

<p>ii. $\angle a + \angle b + \angle c + \angle d = 360^\circ$. (Angles at a point)</p>	
<p>ii. $\angle a + \angle b + \angle c + \angle d = 180^\circ$. (Adjacent Angles on a straight line)</p>	
<p>iv. $\angle a = \angle b$. (Vert. Opposite angles equal)</p>	
<p>vi. $\angle a + \angle b + \angle c = 180^\circ$. (Angles sum of triangle.)</p>	
<p>v. $\angle c = \angle a + \angle b$ (ext. angle of a triangle)</p>	
<p>vi. $\triangle ABC$ is an isosceles \triangle $\Leftrightarrow AB = AC$ (Base sides of \triangle). $\Leftrightarrow \angle b = \angle c$ (Bases \angles of \triangle)</p>	
<p>vii. $\triangle ABC$ is an equilateral \triangle $\Leftrightarrow AB = BC = AC$ (sides of \triangle are equal). $\Leftrightarrow \angle a = \angle b = \angle c = 60^\circ$ (\angles of \triangle are equal).</p>	

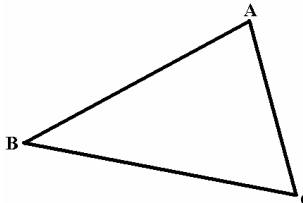
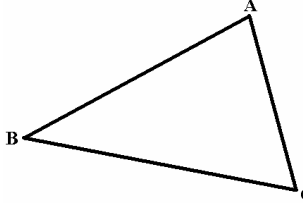
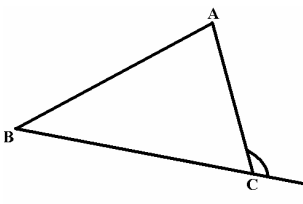
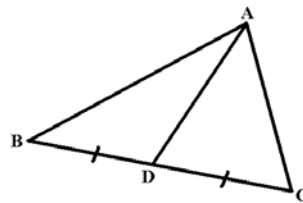
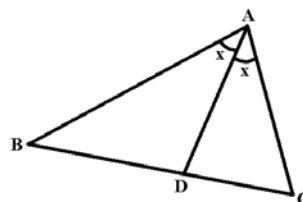
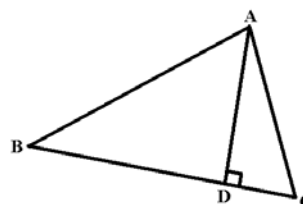
3. Congruent Triangles 全等三角形.

<p>i. $AB = DE$ and $BC = EF$ and $AC = DF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (SSS)</p>	
<p>ii. $AB = DE$ and $BC = EF$ and $\angle ABC = \angle DEF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (SAS)</p>	
<p>iii. $\angle ABC = \angle DEF$ and $BC = EF$ and $\angle ACB = \angle DFE$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (ASA)</p>	
<p>iv. $\angle ABC = \angle DEF$ and $\angle ACB = \angle DFE$ and $AC = DF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (AAS)</p>	
<p>v. $\angle ABC = \angle DEF = 90^\circ$ and $AC = DF$ and $BC = EF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (RHS)</p>	

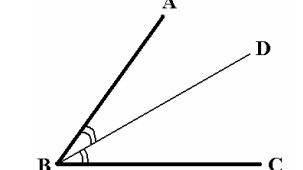
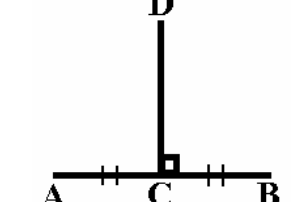
4. Similar Triangles 相似三角形.

<p>i. $\angle ABC = \angle DEF$ and $\angle ACB = \angle DFE$ and $\angle BAC = \angle EDF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (AAA)</p>	
<p>ii. $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \Leftrightarrow \triangle ABC \cong \triangle DEF$ (3 sides proportional)</p>	
<p>iii. $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle BAC = \angle EDF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (ratio of 2 sides, included \angle)</p>	
<p>iv. $\angle ABC = \angle DEF$ and $\angle BAC = \angle EDF$ $\Leftrightarrow \triangle ABC \cong \triangle DEF$ (A.A. Similar)</p>	

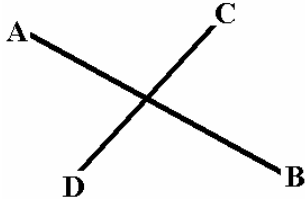
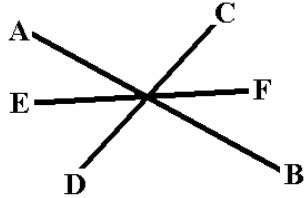
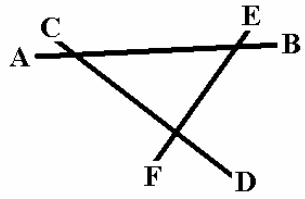
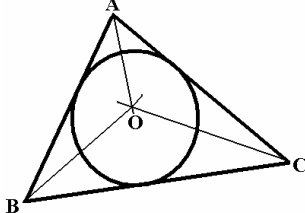
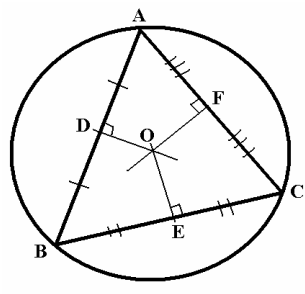
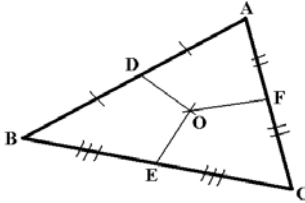
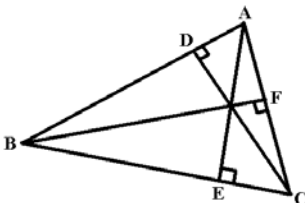
5. Theorem between the sides and angles in a triangle 三角形角度和邊長的定理

<p>i. $AB + BC > AC$ (Triangle Inequality 三角不等式)</p>	
<p>ii. $AB > BC$ $\Leftrightarrow \angle ACB > \angle BAC$ (Bigger interior \angle, Longer corr. sides)</p>	
<p>iii. $\angle ACD > \angle ABC$ and $\angle ACD > \angle BAC$ (ext. $\angle >$ corresponding Interior \angle)</p>	
<p>iv. $BD = DC$ $\Leftrightarrow AD$ is the median of $\triangle ABC$.</p>	
<p>v. $\angle BAD = \angle CAD$ $\Leftrightarrow AD$ is the angle bisector of $\angle A$ in $\triangle ABC$.</p>	
<p>vi. $AD \perp BC$ $\Leftrightarrow AD$ is the altitude of $\angle A$ in $\triangle ABC$.</p>	

6. Bisectors 平分線

<p>i. BD is the angle bisector of $\angle ABC$ $\Leftrightarrow \angle ABD = \angle DBC$</p>	
<p>ii. CD is the perpendicular bisector of AB. $\Leftrightarrow AC = BC$ and $\angle ACD = 90^\circ$.</p>	

7. Point of Intersection of straight lines 直線相交點之特性

<p>i. I is the point of intersection of AB and CD.</p>	
<p>ii. AB, CD and EF are concurrent at point I.</p>	
<p>iii. AB, CD and EF are not concurrent.</p>	
<p>iv. O is the In centre of $\triangle ABC$ \Leftrightarrow AO, BO and CO are the angle bisector of $\angle A$, $\angle B$ and $\angle C$ respectively.</p>	
<p>v. O is the Circumcenter of $\triangle ABC$ \Leftrightarrow DO, EO and FO are the perpendicular bisector of AB, BC and AC respectively.</p>	
<p>vi. O is the Centroid of $\triangle ABC$ \Leftrightarrow DO, EO and FO are the perpendicular bisector of AB, BC and AC respectively.</p>	
<p>vii. O is the Orthocenter of $\triangle ABC$ \Leftrightarrow DO, EO and FO are altitude of $\triangle ABC$.</p>	

(B) Method for solving Geometry questions 解幾何問題方法

1. There are mainly two methods for solving Geometry questions.

解幾何問題主要有兩個方法.

i. Bottom-up Approach (由下至上法).

ii. Association Approach (聯想法).

iii. Applying the answers from the previous parts (應用之前部份的答案).

黎 Sir 提提你  :

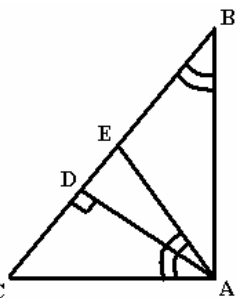
1. Bottom-up Approach (由下至上法):

2. Association Approach (聯想法): _____

3. Applying the answers from the previous parts (應用之前部份的答案):

4. 條條大道通羅馬, 殊途同歸: _____

5. Deductive Geometry(演繹幾何) \Leftrightarrow _____



e.g. 1 例子一

In the figure above, prove that $AD^2 = BC \times CE - DC^2$

(5 marks)

在上圖，証明 $AD^2 = BC \times CE - DC^2$

(5 分)



黎 sir 教你諗:

To prove 証明: $AD^2 = BC \times CE - DC^2$

即係...
即係...
即係...

想起

$$\frac{AC}{BC} = \frac{CE}{AC}$$

由呢度開始做，做到

$$AD^2 = BC \times CE - DC^2$$

即係...

想起

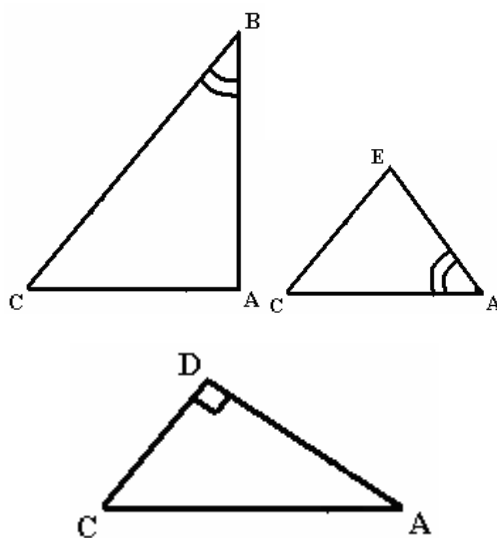
証明

Prove

藍箭嘴: Bottom-up Approach 由下至上法

紅箭嘴: Association Approach 聯想法.

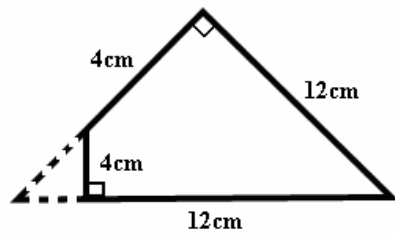
Solutions 題解:



$$AD^2 = BC \times CE - DC^2 \text{ ----- (1A)}$$




- 黎 sir 提提你: 1. (side 邊長)² + \Leftrightarrow _____
2. Ratio of sides 邊長比例 \Leftrightarrow _____



e.g. 2 例子二

After teaching 10 lessons, Andy is very hungry and so he eats a sandwich. When he eat a part of the sandwich (as shown in the figure), he find that the sandwich goes bad. So he throws it away. Find out the area of the part of sandwich where Andy ate. (8 marks)
 上完 10 堂後, Andy 好肚餓所以他吃了一件三文治. 當他吃了一部份後, 他發現這三文治原來已經變壞了. 所以他便棄掉它. 試找出給 Andy 吃了的那一部份三文治的面積. (8 分)



黎 sir 教你諗:

To find 找: Area of $\triangle CDE$

==

and _____

== and _____

== and _____

== and _____

↓ 即係...

想起 _____

想起: _____

(_____)

由呢度開始做, 做到

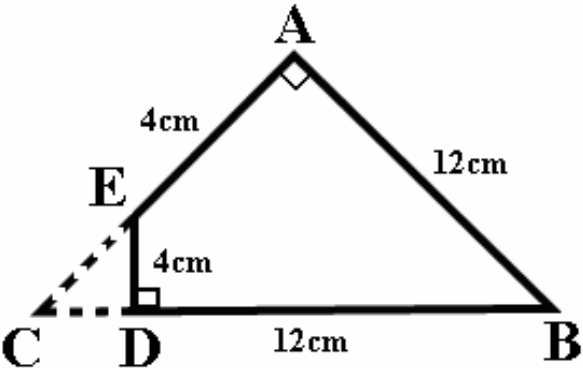
Area of the $\triangle CDE$

證明 _____

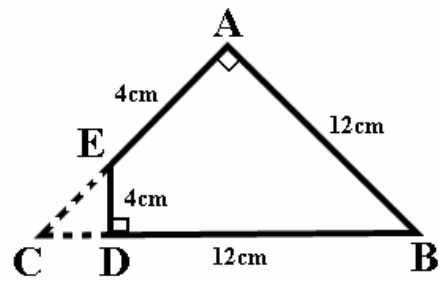
Prove _____

藍箭嘴: Bottom-up Approach 由下至上法

紅箭嘴: Association Approach 聯想法.



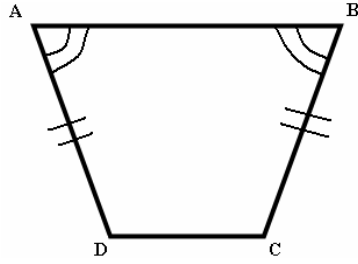
Solutions 題解:



黎 sir 提提你:

1. 找 Δ 面積, 一定想起 Δ 面積公式:

2. 找 Δ 邊長, 有兩個 Δ 有邊相等 \Leftrightarrow _____



e.g. 3 例子三

In the figure above, prove that $\angle ADC = \angle BCD$.

(6 marks)

在上圖，証明 $\angle ADC = \angle BCD$.

(6 分)

黎 sir 教你諗:

To prove 証明:

想起 _____

欠 _____

\therefore 想起証明 _____

由呢度開始做，做到 $\angle ADC = \angle BCD$

證明 _____

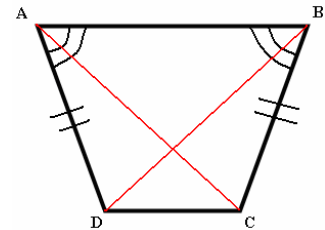
Prove _____

即係... 即係... 即係...

藍箭嘴: Bottom-up Approach 由下至上法

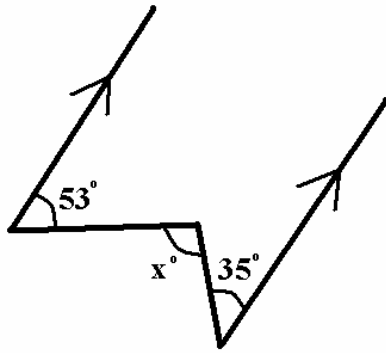
紅箭嘴: Association Approach 聯想法.

Solutions 題解:



黎 sir 提提你:

1. 找 Δ 邊長，有兩個 Δ + 有邊相等 \Leftrightarrow _____



e.g. 4 例子四

Find x . 找 x .

(3 marks / 3分)

黎 sir 教你諗:

想起 _____

想起 _____

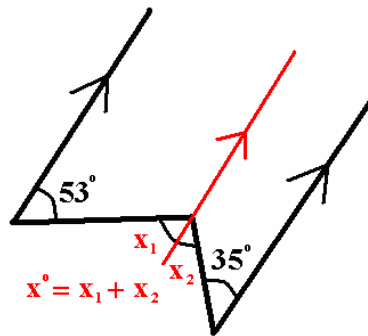
Find 找: x

即係... 即係... 即係...

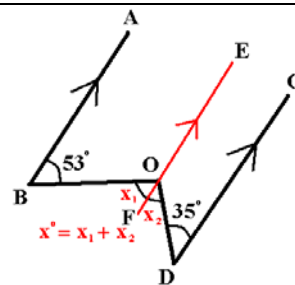
由呢度開始做, 做到 $x = ?$

加 _____
Adding _____

藍箭嘴: Bottom-up Approach 由下至上法 紅箭嘴: Association Approach 聯想法.

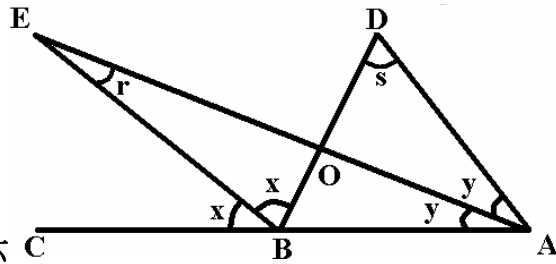


Solutions 題解:



黎 sir 提提你:

1. 有平衡線, 找 $\sphericalangle \Leftrightarrow$ _____



e.g. 6 例子六 C

In the figure, ABC, AOE and BOD are straight lines and $x - y = 50^\circ$. Find $r + s$. (3 marks)

在上圖, ABC, AOE 和 BOD 是直線和 $x - y = 50^\circ$. 求 $r + s$.

(3 分)

黎 sir 教你諗:

想起 _____

想起 _____

Find 找: $r + s$

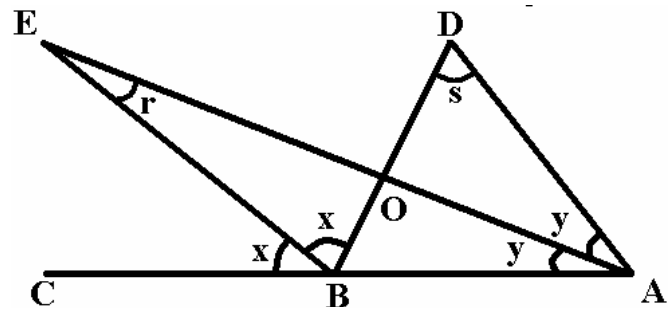
即係... 即係... 即係... 即係...

由呢度開始做, 做到 $r + s = ?$

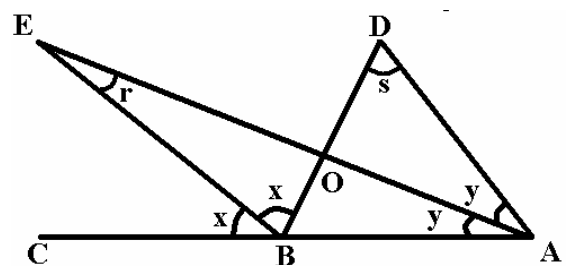
r, s _____

r, s _____

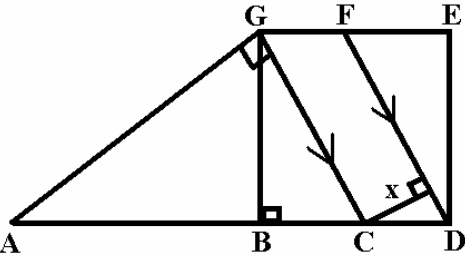
藍箭嘴: Bottom-up Approach 由下至上法 紅箭嘴: Association Approach 聯想法.



Solutions 題解:



- 黎 sir 提提你:**
1. 有外角(ext. \angle), _____
 2. 題目已知既, _____



e.g.7 例子七 A

在上圖中, $AD \perp BC$, $AG \perp GC$, $x \perp DF$, $CG \parallel DF$, BDEG 是正方形. $AB=8$, $BC=2$. 求 x . (8 分)

In the figure, $AD \perp BC$, $AG \perp GC$, $x \perp DF$, $CG \parallel DF$, BDEG is a square. $AB=8m$ $BC=2$. Find x . (8 marks)



黎 sir 教你諗.

Find 找: x

即係...

想起

⇒

想起

⇒

即係...

即係...

即係...

即係...

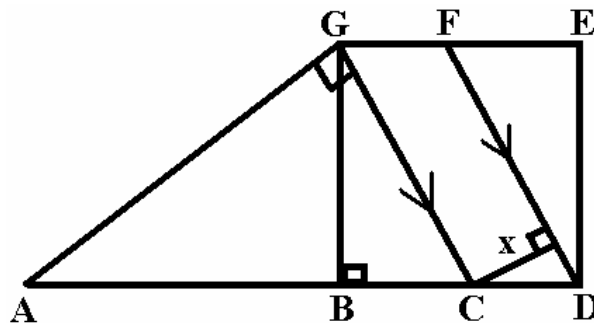
由呢度開始做, 做到 $x = ?$

Solving _____

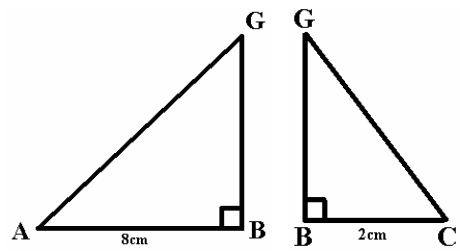
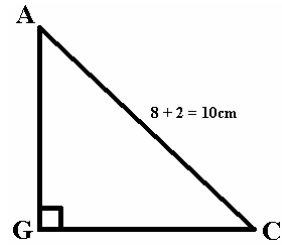
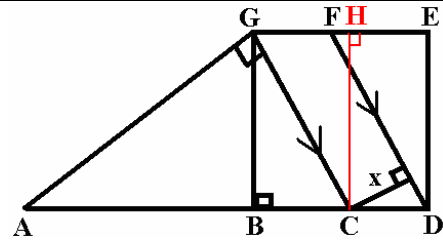
⇒ 解方程!

藍箭嘴: Bottom-up Approach 由下至上法


紅箭嘴: Association Approach 聯想法.

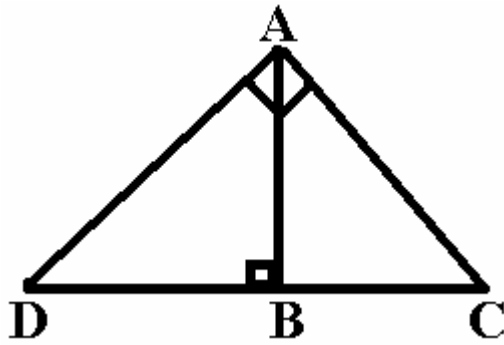


Solutions 題解:



黎 sir 提提你:

1. 平行四邊形的高 \Rightarrow _____
2. 平行四邊形 \Rightarrow _____
3.  \Rightarrow _____
4. 3 equations 方程, 3 unknowns 變數 \Rightarrow _____
5. 最後答案 \Rightarrow _____



e.g.8 例子八

In the figure, $CD \perp AB$, $\angle ACB = 90^\circ$, $AD = 5$
 在上圖中, $CD \perp AB$, $\angle ACB = 90^\circ$, $AD = 5$

黎sir 教你認:

CD, AD \Rightarrow _____

DB \Rightarrow _____

Find 找: _____ 即係...

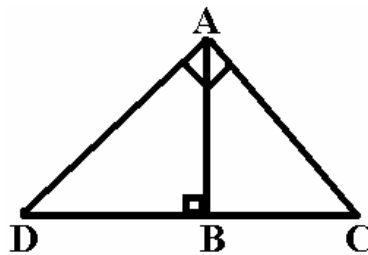
即係...

由呢度開始做, 做到 $CD = ?$

Prove 證明 _____

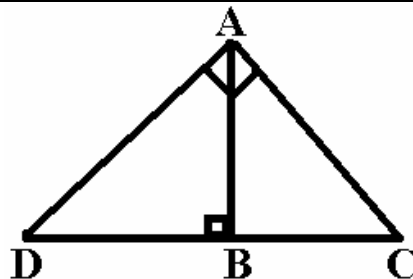


藍箭嘴: Bottom-up Approach 由下至上法 紅箭嘴: Association Approach 聯想法.



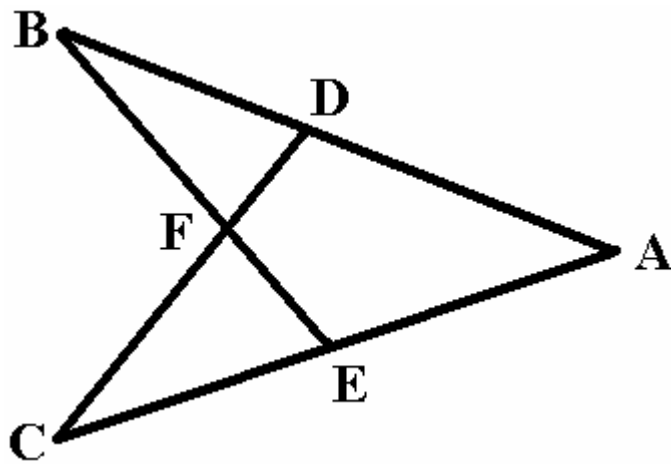
Solutions 題解:

Consider $\triangle ACD$ and $\triangle BAD$



黎sir 提提你:

1. 找 \triangle 邊長, 有兩個 \triangle 有邊相等 \Leftrightarrow _____




e.g. 9 例子九

In the figure, $AB=AC$, $AD=AE$, Prove $BF=CF$.

(3 marks)

在上圖中, $AB=AC$, $AD=AE$, 証明 $BF=CF$.

(3 分)



黎 sir 教你諗:

Prove 証明: _____

即係...

即係...

即係...

想起: _____

(_____)

$BF \Rightarrow$ _____

$CF \Rightarrow$ _____

想起: _____

(_____)

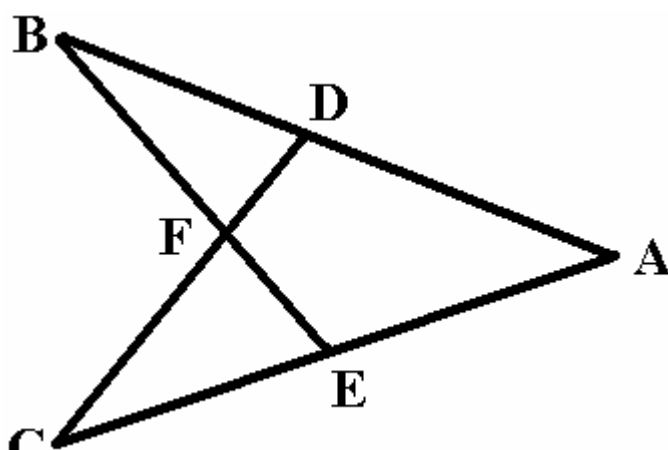
$\angle DBF, AB \Rightarrow$ _____

$\angle ECF, AC \Rightarrow$ _____

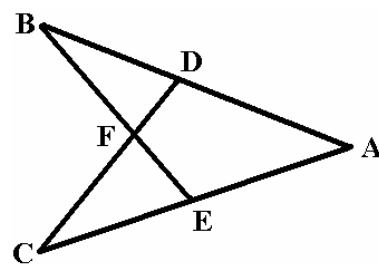
由呢度開始做, 做到 $BF = CF$

Prove 証明 _____

藍箭嘴: Bottom-up Approach 由下至上法 **紅箭嘴: Association Approach 聯想法.**

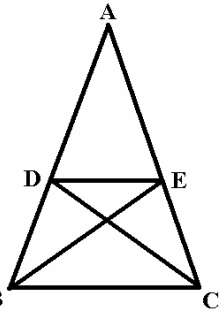


Solutions 題解:



黎 sir 提提你:

1. 找 Δ 邊長, 有兩個 Δ 有邊相等 \Leftrightarrow _____



e.g. 10 例子十

In the figure, $AB=AC$, $\angle ABE = \angle ACD$. Prove that $AD = AE$.

(2 marks)

在上圖中, $AB=AC$, $\angle ABE = \angle ACD$. 證明 $AD = AE$.

(2 分)



黎 sir 教你諗:

想起: _____

(_____)

$AD \Rightarrow$ _____

$AE \Rightarrow$ _____

由呢度開始做, 做到 $AD = AE$

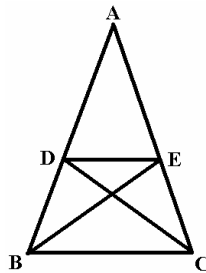
Prove 證明: _____ 即係...

Prove 證明 _____

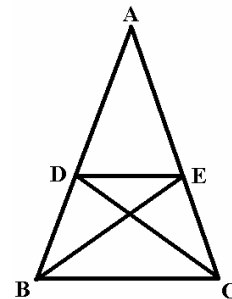


藍箭嘴: Bottom-up Approach 由下至上法

紅箭嘴: Association Approach 聯想法.

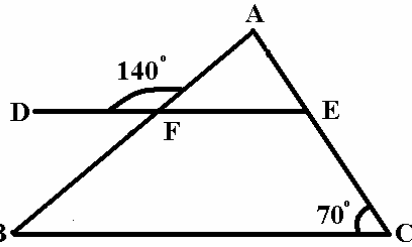


Solutions 題解:



黎 sir 提提你:

1. 找 Δ 邊長, 有兩個 Δ + 有邊相等 \Leftrightarrow _____



e.g. 11 例子十一

In the figure, $AB=BC$. Prove $BC \parallel DE$.

(4 marks)

在上圖中, $AB=BC$. 證明 $BC \parallel DE$.

(4 分)

黎 sir 教你諗:

想起: _____
(_____)

Prove 證明: $BC \parallel DE$ 即係...
即係...

由呢度開始做, 做到 $BC \parallel DE$

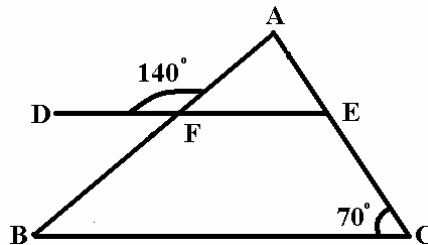
找 _____, _____

$\angle EFB \Rightarrow$ _____
 $\angle CBF \Rightarrow$ _____

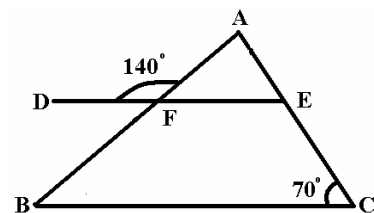


藍箭嘴: Bottom-up Approach 由下至上法

紅箭嘴: Association Approach 聯想法.

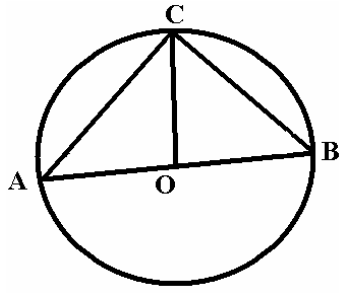


Solutions 題解:



黎 sir 提提你:

1. 找平行邊 \Leftrightarrow _____



e.g. 12 例子十二

In the figure, O is the center of the circle. Prove $\angle ACB = 90^\circ$ (4 marks)
 在上圖中, O 是圓形的圓心. 證明 $\angle ACB = 90^\circ$.



黎 sir 教你諗:

圓形半徑 Δ

\Leftrightarrow _____

Prove 證明: $\angle ACB = 90^\circ$

即係...
 即係...
 即係...

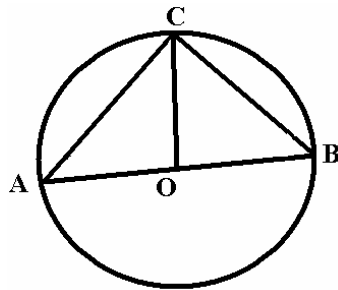
由呢度開始做, 做到 $\angle ACB = 90^\circ$

考慮 _____, _____

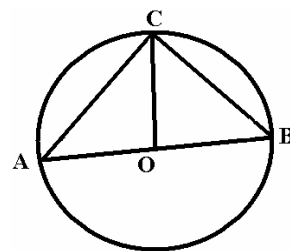


藍箭嘴: Bottom-up Approach 由下至上法

紅箭嘴: Association Approach 聯想法.



Solutions 題解:



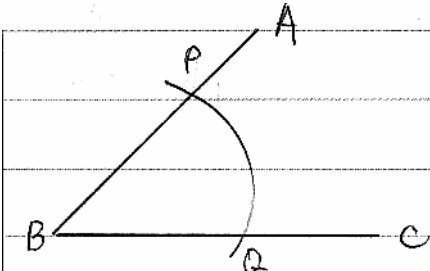
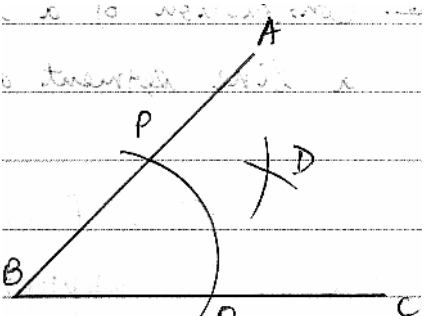
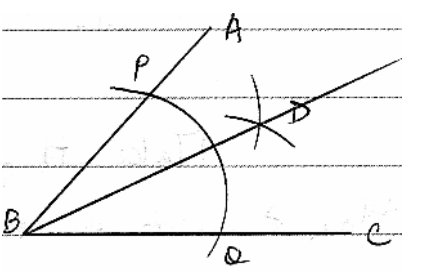
黎 sir 提提你:

1. 圓形半徑 Δ \Leftrightarrow _____

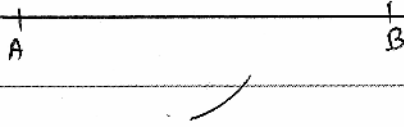
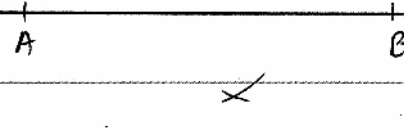
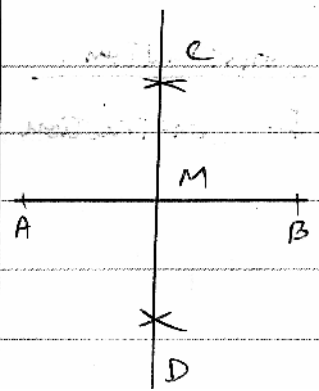
2. 找 Δ 內角 \Leftrightarrow _____

(C) Construction of Geometry 幾何繪圖

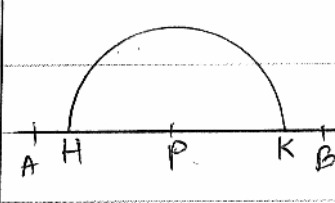
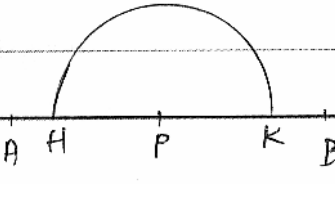
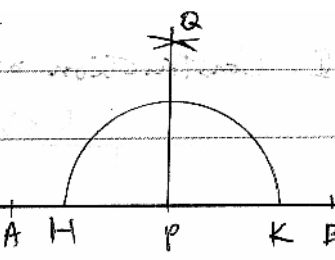
1. Construction of Angle bisector of an angle.

Step 1	Take B as the center. Choose an appropriate radius, and draw an arc to intersect BA and BC at P and Q respectively.	
Step 2	Take P and Q as centers, select a radius longer than $\frac{1}{2}PQ$ and draw two arcs such that these two arcs intersect at D.	
Step 3	Join BD. BD is the angle bisector of $\angle ABC$	

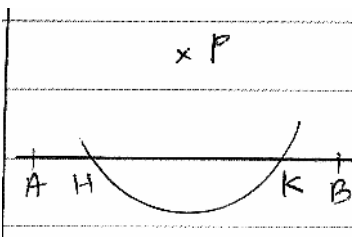
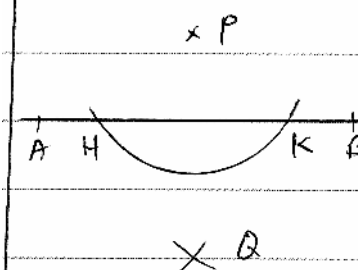
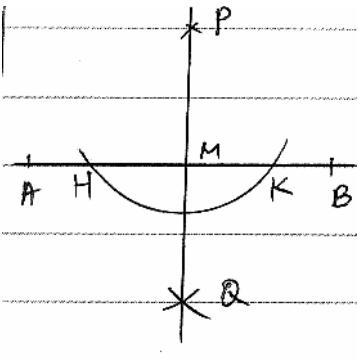
2. Construction of perpendicular bisector of a line segment.

Step 1	Take A as the center. Choose a radius longer than $\frac{1}{2}AB$ and draw an arc above and below the line segment AB.	
Step 2	Take B as the center. Use the same radius and draw another two ones to intersect the arcs at C and D.	
Step 3	Join CD. CD intersects AB at M, M is the mid-point of AB. CD is the perpendicular bisector of AB	

3. Construction of a line passing through a given point on a line segment and perpendicular to that line segment.

Step 1	Take P as the center. Choose an appropriate radius and draw an arc to intersect AB at H and K.	
Step 2	Take H and K as centers. Use a radius longer than $\frac{1}{2}HK$ and draw two arcs such that they meet at Q.	
Step 3	Join PQ. Then $PQ \perp AB$.	

4. Construction of a line passing through a point lying outside a line segment and perpendicular to that line segment.

Step 1	Take P as the center. Choose an appropriate radius and draw an arc to intersect AB at H and K.	
Step 2	Take H and K as centers. Choose a radius longer than $\frac{1}{2}HK$ and draw two arcs such that the two arcs meet at Q.	
Step 3	Join PQ. The line PQ intersects AB at M, and $PQ \perp AB$	

This is the blank page.