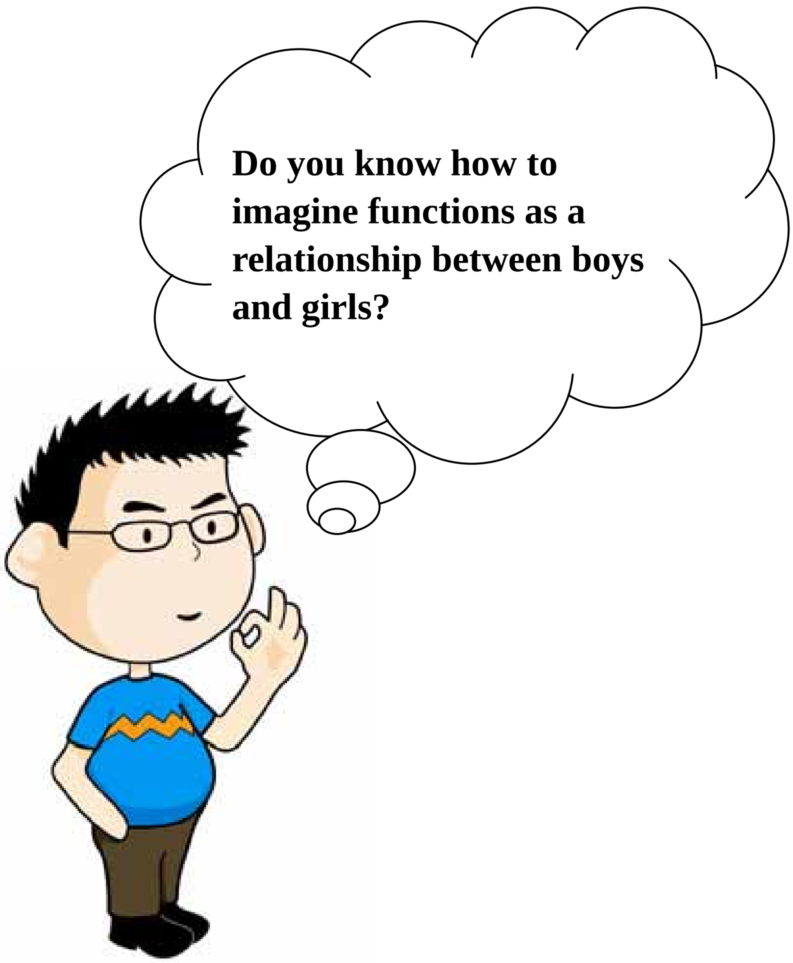
Mathematics

數學科

GCE Mathematics:
Algebra and Functions
Paper C2 and C3



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Functions: Paper C2

1. Relationship between Dividend, Divisor, Quotient and Remainder.

Andy wants to point out that :

f(x)	=	**(x-a) / (ax-b)**	×	**Q(x)**	+	**R(x)**
Dividend	=	**Divisor**	×	**Quotient**	+	**Remainder**

e.g. $(x^2+3)\div(x-1)$, $(x^2+3)=(x-1)(x+1)+4$ **[Long Division]**

e.g. $(2x^2+3)\div(2x-1)$, $(2x^2+3)=(2x-1)(x+\frac{1}{2})+\frac{7}{2}$ **[Long Division]**

2. Remainder Theorem: Find Remainder of $f(x)$.

Remainder Theorem

When a polynomial $f(x)$ is divided by $x-a$, the remainder is equal to $f(a)$.

When a polynomial $f(x)$ is divided by $mx-n$, the remainder is equal to $f\left(\frac{n}{m}\right)$.

Andy wants to point out that :

1. Remainder Theorem $\Rightarrow$ Find Remainder

2. Divided by x-a, put x = a in f(x) **Easy to Remember: (x-a=0, x=a)**

3. Divided by mx-n, put x = n/m in f(x) **Easy to Remember: (mx-n=0, x=n/m)**

3. Factor Theorem: Find Factor of f(x)

Factor Theorem

$f(x)$ **is a polynomial and** $f(a)=0 \Leftrightarrow x-a$ **is a factor of the polynomial** $f(x)$.

$f(x)$ **is a polynomial and** $f\left(\frac{n}{m}\right)=0 \Leftrightarrow mx-n$ **is a factor of the polynomial** $f(x)$.

Andy wants to point out that :

1. Factor Theorem $\Rightarrow$ Find Factor (linear factor) $\Rightarrow$ By Guessing Only !!!

2. Meaning of a Factor: The remainder after division equals zero.

3. Factor Theorem is a special case of Remainder Theorem. i.e. the remainder = zero !!!

4. To Solve polynomial equations with degree = 3

Q: How to solve polynomial equations with degree = 3

A: Factor Theorem + Long Division + Factorization of quadratic polynomials.

Functions: Paper C3

1. Simplification of the following fractions:

$\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.

2. Definition of a function: $y = f(x)$ / $y = f : x \to \ldots$

i. $\forall x \in D$, $\exists y \in R$ **s.t. f(x) = y**

(For all x belonging to the Domain, there exists a y belonging to the Range such that f(x) = y)

ii. $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

(Same x => Same y)

iii. Domain: x-restrictions / Range: y-restrictions

Andy wants to point out that  **:**

1. Definition of Function => Imagine the relationship between boys and girls.

=> x: Boys, y: Girls

i. $\forall x \in D$, $\exists y \in R$ **s.t. f(x) = y**

(i.e. Every boy (x) have to chase after girls (y) ! 所有男仔(x) 都要追女仔(y) !)

ii. $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

(i.e. One boy(x) can chase after one girl(y) only! 一個男仔(x) 只可以追一個女仔(y)！)

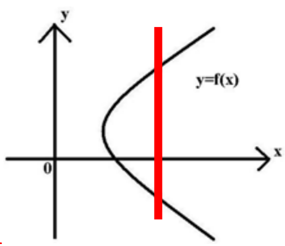
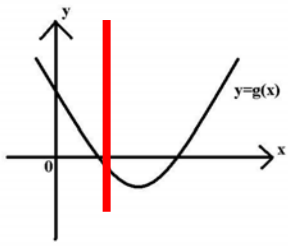
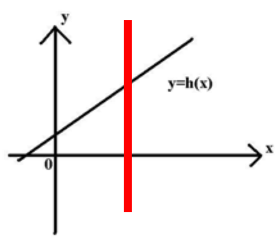
3. How to check whether it is a function?

Q: How to check whether is is a function.

A: If one x is mapped to 2 values of y, then it is not a function

Andy wants to point out that  **:**

1. Easy to Remember: Vertical Line Test on the graph.

 **e.g. 1**	 **e.g. 2**	 **e.g. 3**
$\because$ **2 intersection points.**	$\because$ **1 intersection point.**	$\because$ **1 intersection point.**
$\therefore f(x)$ **is <u>NOT</u> a function.**	$\therefore f(x)$ **is a function.**	$\therefore f(x)$ **is a function.**

4. One-to-One mapping and Many-to-one mapping

Q: What is one-to-one mapping from domain to range?

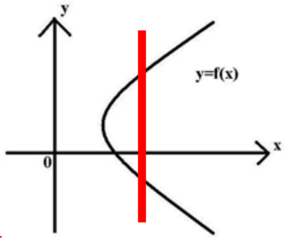
A: Same y $\Rightarrow$ Same x

Q: What is many-to-one mapping from domain to range?

A: Same y $\Rightarrow$ More than one x

Andy wants to point out that :

1. Easy to Remember: Horizontal Line Test on the graph.

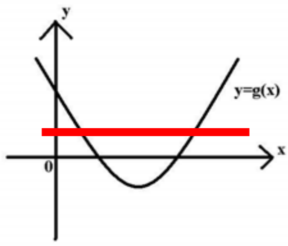


e.g. 1

$\because$ $f(x)$ **is not a function.**

(even V.L.T. can't be passed !!!)

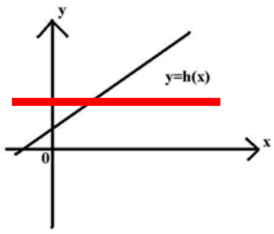
$\therefore$ **We can ignore it.**



e.g. 2

$\because$ **2 intersection points**

$\therefore$ $f(x)$ **is a <u>many-to-one</u> mapping**



e.g. 3

$\because$ **1 intersection point.**

$\therefore$ $f(x)$ **is a <u>one-to-one</u> function.**

5. Composite Functions

Q: What is composite functions $fg(x)$?

A: $fg(x)$ means "Do g first then f "

6. Inverse function f^{-1}

Q: What is inverse functions f^{-1}?

A: Reserve the relationship of x and y.

Andy wants to point out that :

1. Easy to Remember: Change of subject (i.e. x in terms of y)

2. Domain of f = Range of f^{-1}

3. Range of f^{-1} = Domain of f

4. $ff^{-1}(x) = f^{-1}f(x) = x$

5. Pay attention to find the Domain and Range of $f(x)$.

$f(x)=\frac{1}{1-x}$, $x \in R, x \neq 1$ $\Rightarrow f(x) \in R$	$y=\frac{1}{x}$, $x \in R, x \neq 0$ $\Rightarrow f(x) \in R$	$y=(x-1)^2$, $x \in R$ $\Rightarrow f(x) \in R, f(x) \geq 0$

6. Standard procedures for solving functions questions:

a. Find the range of a given $f(x)$ with domain.

b. Find the inverse function $f^{-1}(x)$ with domain and Range specified.

c. Sketch the function. (refer to point 7)

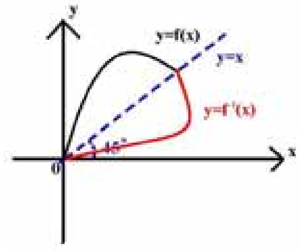
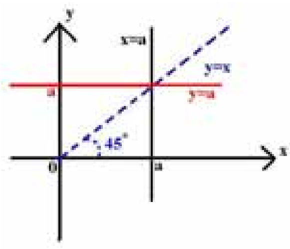
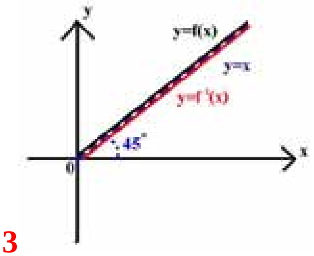
7. Symmetric Properties between $f(x)$ and $f^{-1}(x)$ about the $y=x$

Q: What is the relationship between $f(x)$ and $f^{-1}(x)$ on the graph?

A: They are symmetrical about $y=x$.

Andy wants to point out that :

1. Easy to Remember: $f(x)$ and $f^{-1}(x)$ are symmetrical about $y=x$.

 **e.g. 1**	 **e.g. 2**	 **e.g. 3**
$y=x$ **: Dotted straight line**	$y=x$ **: Dotted straight line**	$y=x$ **: Dotted straight line**
$f(x)$ **: curve black in-color**	$f(x)$ **: curve black in-color**	$f(x)$ **: curve black in-color**
$f^{-1}(x)$ **: curve red-in-color**	$f^{-1}(x)$ **: curve red-in-color**	$f^{-1}(x)$ **: curve red-in-color**

8. The meaning of modulus.

Q: What is the meaning of modulus?

A: Modulus means "make every thing positive".

Strictly Speaking: $|a| = \begin{cases} a & if \ a \geq 0 \\ -a & if \ a < 0 \end{cases}$

e.g.1 $|7| = 7$

e.g.2 $|-7| = -(-7) = 7$

Andy wants to point out that :

1. Easy to Remember: $|a| = a$ if a is positive, $|a| = -a$ if a is negative.

(如果 a 是正數⇒不變，如果 a 是負數⇒比個負號佢．)

9. The modulus functions $y=|ax+b|$, $|f(x)|$ and $f(|x|)$

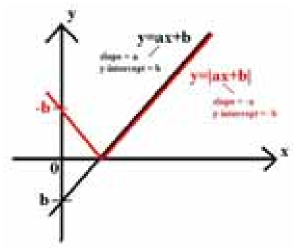
Q: What is the meaning of modulus function $y=|ax+b|$?

A: $y=|ax+b|$ means "make all y-coordinates <u>on the straight line</u> become positive"

Andy wants to point out that :

1. Easy to Remember: Change all "–y" to "+y".

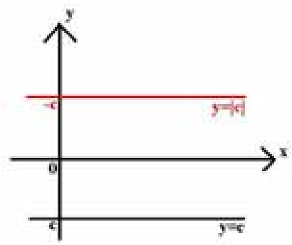
(將所有 "–y" 變 "+y".)



e.g. 1

$y=ax+b$ **: black in-color**

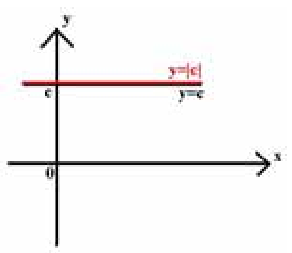
$y=|ax+b|$ **: red-in-color**



e.g. 2

$y=ax+b$ **: black in-color**

$y=|ax+b|$ **: red-in-color**



e.g. 3

$y=ax+b$ **: black in-color**

$y=|ax+b|$ **: red-in-color**

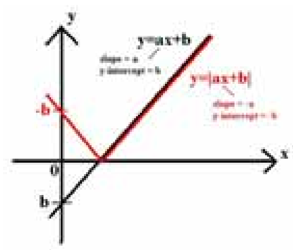
Q: What is the meaning of modulus function $|f(x)|$?

A: $|f(x)|$ means "make all y-coordinates <u>on the curve/straight line</u> become positive"

Andy wants to point out that :

1. Easy to Remember: Reflect all "–y" side about x-axis and then delete all "-y" side

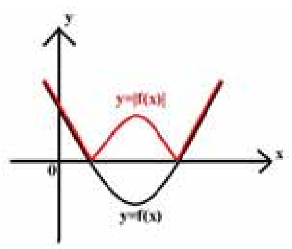
(將所有"–y"變反上去變"+y", 再消滅所有"–y")



e.g. 1

$y=ax+b$ **: black in-color**

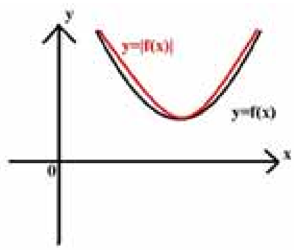
$y=|ax+b|$ **: red-in-color**



e.g. 2

$y=f(x)$ **: black in-color**

$y=|f(x)|$ **: red-in-color**



e.g. 3

$y=f(x)$ **: black in-color**

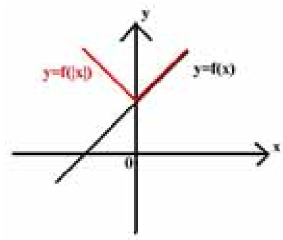
$y=|f(x)|$ **: red-in-color**

Q: What is the meaning of modulus function $y = f(|x|)$?

A: $y = f(|x|)$ means "make all y-coordinates of negative x-coordinates become the y-coordinates of corresponding positive x-coordinates"

Andy wants to point out that  **:**

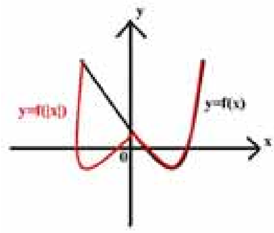
1. Easy to Remember: Delete all "–x" side and then reflect all "+x" side about y-axis.
(消滅所有"–x", 再將所有"+x"反過去變"-x",)



e.g. 1

$y = f(x)$ **: black in-color**

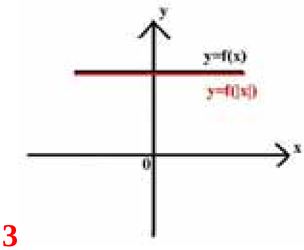
$y = f(|x|)$ **: red-in-color**



e.g. 2

$y = f(x)$ **: black in-color**

$y = f(|x|)$ **: red-in-color**



e.g. 3

$y = f(x)$ **: black in-color**

$y = f(|x|)$ **: red-in-color**

10. Transformation of functions $y = \pm c \bullet f(\pm ax \pm b) \pm d$.

$$y = \pm c \bullet f(\pm ax \pm b) \pm d$$

$\pm d \Leftrightarrow$ 上/下移動 **Shift upwards / downwards**

$\pm b \Leftrightarrow$ 左/右移動 **Shift to left / right**

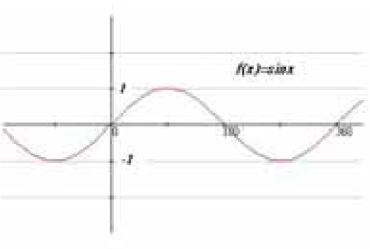
$0 < +a < 1$，$+a > 1 \Leftrightarrow$ 拉長/夾扁 **scaling**

$-a \Leftrightarrow$ 左右反轉 **reflect about y-axis**

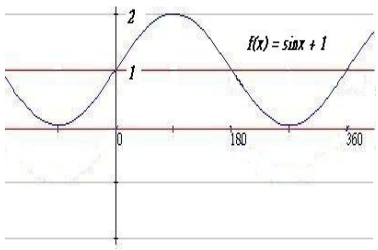
$+c > 1$，$0 < +c < 1 \Leftrightarrow$ 拉高/壓扁 **scaling**

$-c \Leftrightarrow$ 上下反轉 **reflect about x-axis**

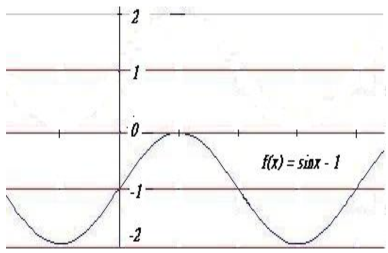
$f(x) = \sin x$



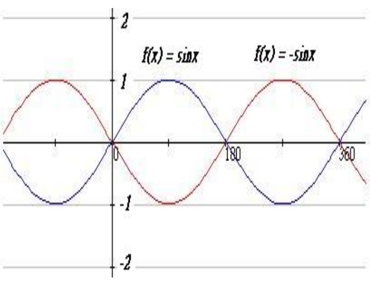
$f(x) = \sin x + 1$ 上移 ↑



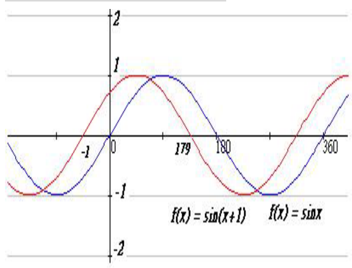
$f(x) = \sin x - 1$ 下移 ↓



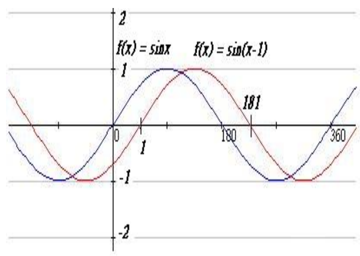
$f(x) = -\sin x$ 上下反轉 ↑↓



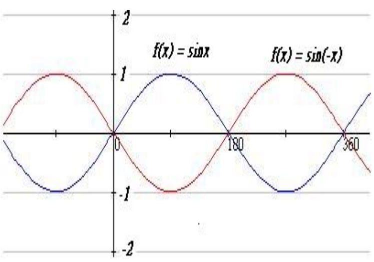
$f(x) = \sin(x+1)$ 左移 ←



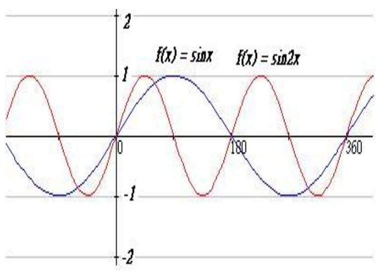
$f(x) = \sin(x-1)$ 右移 →



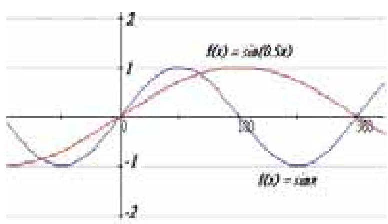
$f(x) = \sin(-x)$ 左右反轉 →←



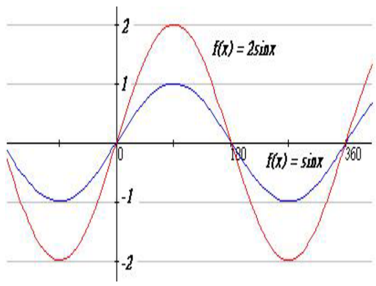
$f(x) = \sin 2x$ 夾扁 →←



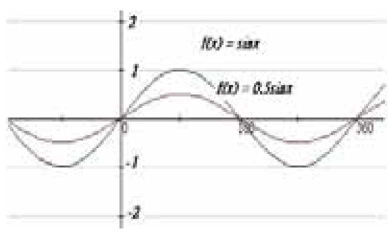
$f(x) = \sin \frac{1}{2} x$ 拉長 ↔



$f(x) = 2\sin x$ 拉高 ↕



$f(x) = \frac{1}{2}\sin x$ 壓扁 ↓↑



Andy wants to point out that  **:**

1. You have to know how to do <u>more than one transformation</u> for a given $f(x)$.

2. For Trigo. Functions, e.g. $4\cos(-x) \Leftrightarrow$ **know to sketch the original** $f(x)$ **too.**

The End.