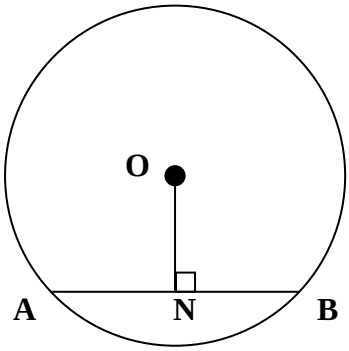
Suggested Solutions for HKCEE Mathematics Test 2:

Deductive Geometry in Circles

Time Limit: 40 minutes **Marks: / 50**

Part 1: Multiple Choices Questions: (20%)

1. In the figure, O is the centre of the circle and radius of the circle is 13 cm. $ON = 5$ cm. Find the length of AB.



A. 10 cm
B. 12 cm
C. 18 cm
D. 24 cm

黎 Sir 提提你  **: (D)**

$\because \Delta$ **ONA is a right-angled** Δ**.**

$\therefore OA^2 = AN^2 + NO^2$

$AN^2 = OA^2 - NO^2$

$AN^2 = 13^2 - 5^2$

$AN = \sqrt{144}$

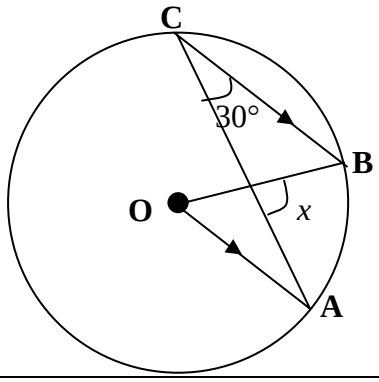
$AN = 12$

$AN = NB = 12$ **(Line from centre perpendicular to chord, bisect chord.)**

$\therefore \quad AB = AN + NB$

$AB = 24cm$

2. In the figure, O is the centre of the circle. Find the value of x.



A. 45°
B. 60°
C. 75°
D. 90°

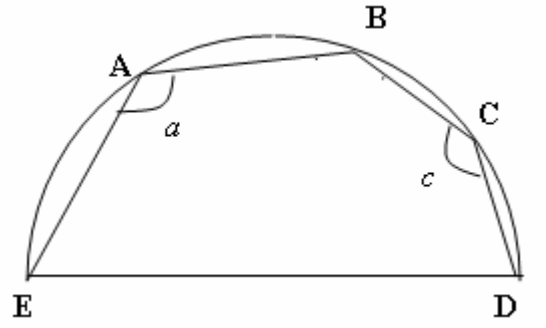
黎 Sir 提提你  **: (D)**

$\angle AOB = 2\angle BCA$ **(∠ at center twice ∠ at circumference)**
$\angle AOB = 60^\circ$

$\angle CBO = \angle AOB$ **(alt ∠s, AO // BC)**
$\angle CBO = 60^\circ$

$\angle ACB + \angle CBO = x$ **(ext.∠ of Δ)**
$x = 30^\circ + 60^\circ$
$x = 90^\circ$

3. The following figure is a semi-circle and DE is a diameter.
Find the value of $a+c$.



A. 180° **B.** 225° **C.** 270° **D.** 350°

黎 Sir 提提你  **: (C)**

Construct a straight line AD respectively.

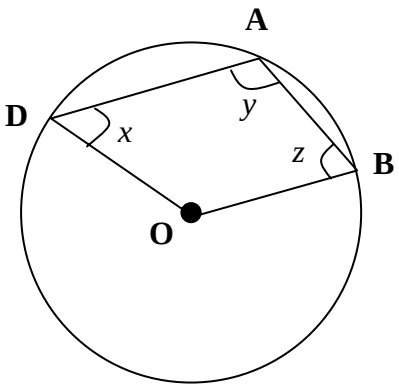
$\angle EAD = 90^\circ$ **(∠ in semi circle)**

$\angle BAD = a - 90^\circ$

$a - 90^\circ + c = 180^\circ$ **(oppo ∠s, cyclic quadrilateral)**

$a + c = 270^\circ$

4. In the figure, O is the centre of the circle, $x + z =$



A. $\frac{1}{2}y$

B. y

C. $2y$

D. $180° - y$

黎 Sir 提提你  **: (B)**

$\because \Delta$ **OAB is a an isosceles** Δ**.**

$\therefore \angle OAB = z$ **(Base ∠s of isosceles** Δ**)**

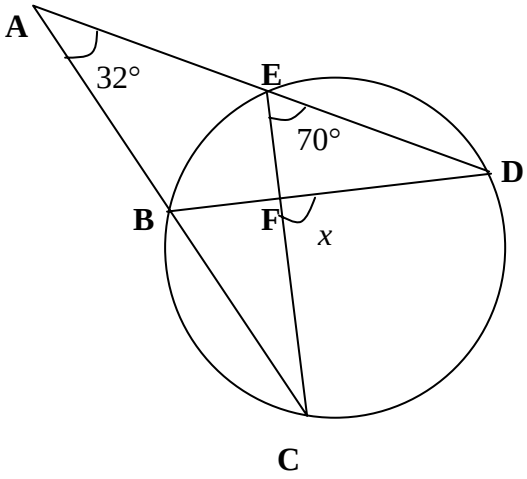
$\because \Delta$ **OAD is a an isosceles** Δ**.**

$\therefore \angle OAD = x$ **(Base ∠s of isosceles** Δ**)**

$y = x + z$

$x + z = y$

5. In the figure, AED and ABC are straight lines, find the value of x.



A. 94°
B. 100°
C. 102°
D. 108°

黎 Sir 提提你 : **(D)**

$\angle ECA = \angle EDB$ **(∠ s in same segment)**

In Δ **ACE,** $\angle FAC + \angle FCA = 70^\circ$ **(ext.∠ of Δ)**

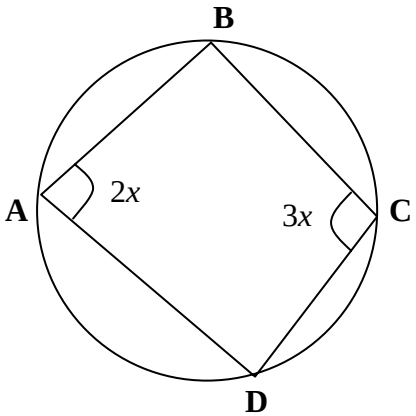
$\angle FCA = 70^\circ - 32^\circ$

$\angle FCA = 38^\circ$

In Δ **EDF,** $70^\circ + 38^\circ = x$ **(ext.∠ of Δ)**

$x = 108^\circ$

6. In the figure, ABCD is a cyclic quadrilateral. If $\angle BAD = 2x$ **and** $\angle BCD = 3x$**, then** $x =$



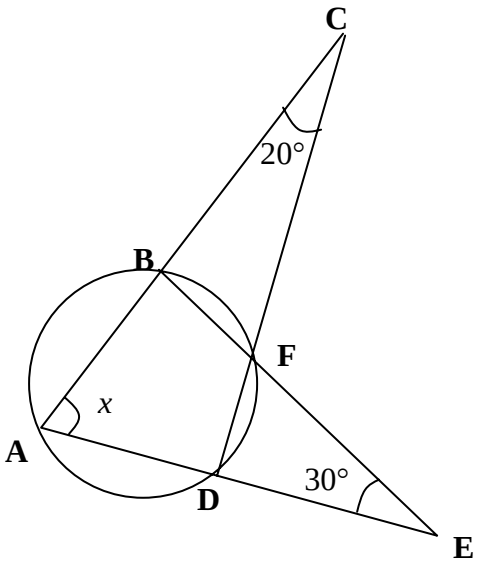
A. 36°
B. 48°
C. 60°
D. 72°

黎 Sir 提提你 : **(A)**

$2x + 3x = 180^\circ$ **(oppo ∠ s, cyclic quadrilateral)**

$x = 36^\circ$

7. In the figure, ABC, ADE, BFE and DFC are straight lines.
Find the value of x.

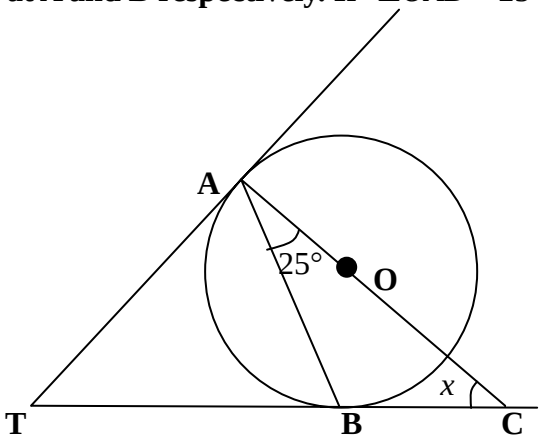


A. 55°
B. 60°
C. 65°
D. 70°

黎 Sir 提提你  **: (C)**

$x+30^\circ=\angle CBF$ **(ext. ∠ Δ)**
$\angle CBF+20^\circ=\angle BFD$ **(ext. ∠ Δ)**
$\angle BFD+x=180^\circ$ **(oppo ∠s, cyclic quadrilateral)**
$x+30^\circ+20^\circ+x=180^\circ$
$x=65^\circ$

8. In the figure, O is the centre of the circle, TA and TB are the tangents to the circle at A and B respectively. If $\angle OAB = 25°$, then $x =$



A. 25°
B. 30°
C. 35°
D. 40°

黎 Sir 提提你  : **(D)**

Construct a line OB s.t. ΔOAB is an isos.Δ.

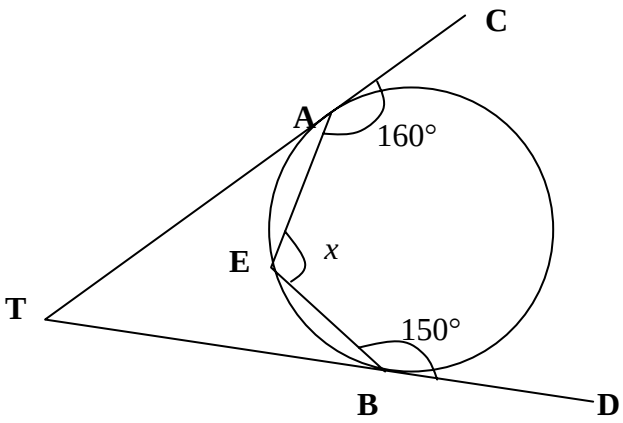
$\angle OAB = 25^\circ$ **(base $\angle$ s of isos. Δ)**

$\angle ABT = 90^\circ - 25^\circ$

$x + 25^\circ = \angle ABT$ **(ext. $\angle$ of Δ)**

$x = 40^\circ$

9. In the figure, TA and TB are tangents. $x =$



A. 100°
B. 110°
C. 120°
D. 130°

黎 Sir 提提你  : **(D)**

Construct a line from B and A to center O respectively.

$\angle OAE = 160^\circ - 90^\circ = 70^\circ$ **and** $\angle OBE = 150^\circ - 90^\circ = 60^\circ$

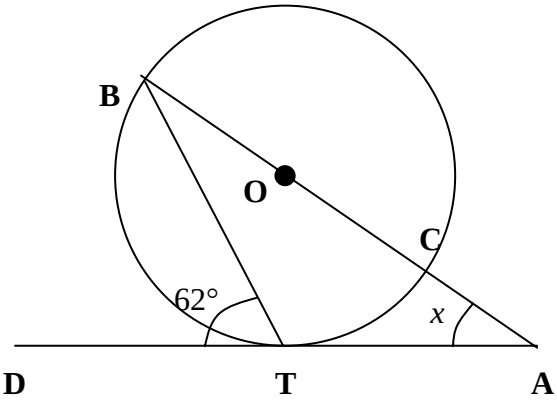
$\angle AOB = 2\angle AEB$ **($\angle$ at center twice of $\angle$ at circumference)**

$y = 360^\circ - 2x$

$x + 360^\circ - 2x + 70^\circ + 60^\circ = 360^\circ$ **($\angle$s sum of polygon)**

$x = 130^\circ$

10. In the figure, TA is a tangent to the circle at T and BC is a diameter produced to meet at A. If $\angle BTD = 62°$, then $x =$



A. 17°
B. 28°
C. 34°
D. 42°

黎 Sir 提提你  **: (C)**

Construct a line OT s.t. ΔOBT is an isos.Δ.

$\angle OTB = 90^\circ - 62^\circ = 28^\circ$

$\angle OBT = \angle OTB = 28^\circ$ **(base $\angle$ s of isos. Δ)**

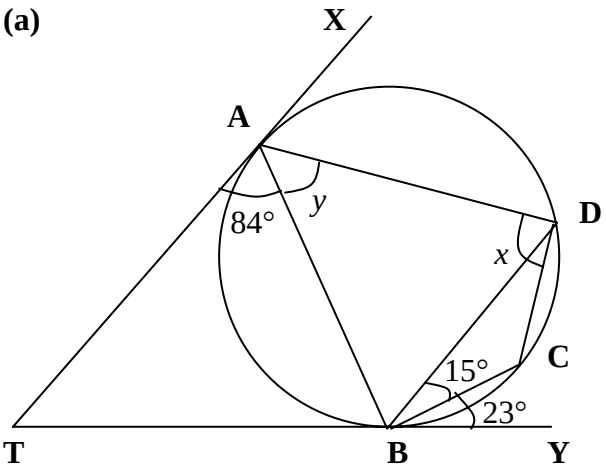
$x + 28^\circ = 62^\circ$ **(ext. $\angle$ of Δ)**

$x = 34^\circ$

Part 2: Structured Questions: (35%)

1. In each of the following figures, TA and TB are the tangents to the circle. O is the centre of the circle. Find the unknown(s).

(a)



(2 marks)

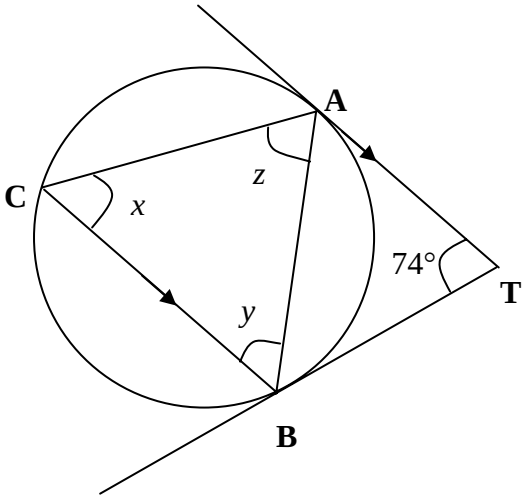
黎 Sir 提提你  :

$\angle TAB = \angle TBA = 84^\circ$ **(Tangent properties)**

$x = \angle TBA = 84^\circ$ **(∠ s in alternate segment)**

$y = \angle DBY = 38^\circ$ **(∠ s in alternate segment)**

(b)



(4 marks)

黎 Sir 提提你  :

$x + y + z = 180^\circ$ **(∠ s sum of Δ)**

$y = \angle TAB$ **(Alt. ∠ s equal, BC // TA)**

$\angle TAB = \angle TBA = y$ **(Tangent properties)**

$$y = \frac{180^\circ - 74^\circ}{2}$$

$y = 53^\circ$

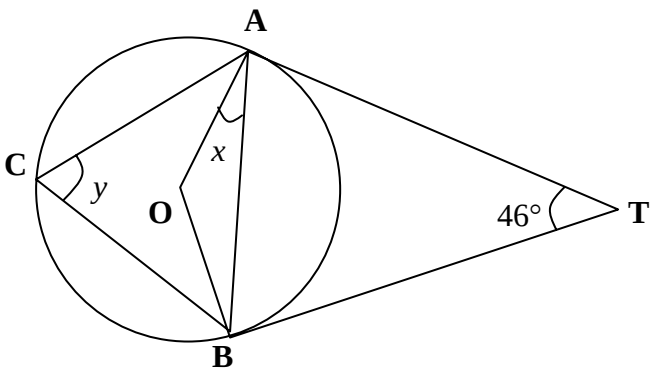
$x = TAB^\circ$ **(∠ in alt. Segment)**

$x = 53^\circ$

$z = 180^\circ - 53^\circ - 53^\circ$

$z = 74^\circ$

(c)



(4 marks)

黎 Sir 提提你  :

$\angle OBA = x$ **(Base ∠ s of isos. Δ)**

$\angle AOB = 180^\circ - 2x$ **(∠ s sum of Δ)**

$\angle AOB = 2y$ **(∠ at center twice ∠ at circumference)**

$\angle TAB = 90^\circ - x$ **and** $\angle TBA = 90^\circ - x$

$\angle TAB + \angle TBA + 46^\circ = 180^\circ$ **(∠ s sum of Δ)**

$90^\circ - x + 90^\circ - x + 46^\circ = 180^\circ$

$x^\circ = 23^\circ$

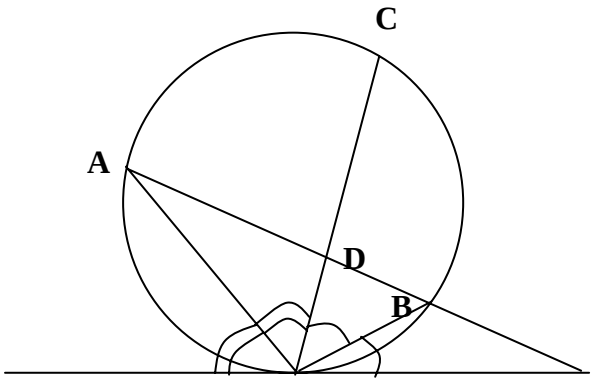
$2y = 180^\circ - 2(23^\circ)$

$y = 67^\circ$

2. In the figure, TQ is the tangent to the circle at P. Chords AB and PC intersect at D such that AP and BP bisect $\angle CPQ$ and $\angle CPT$ respectively.

Prove that (a) AB is a diameter of the circle.

(b) $AB \perp PC$.



(5 marks)

黎 Sir 提提你  :

(a)

$\angle APQ + \angle DPA + \angle DPB + \angle BPT = 180^\circ$ **(∠ s on a straight line)**

$\angle DPA + \angle DPB = 90^\circ$

AB **is the diameter of the circle.**

(b)

$\angle ADC = \angle DAP + \angle DPA$ **--- (1)**

$\angle ADC = \angle DBP + \angle DPB$ **--- (2)**

By (1) + (2) gives

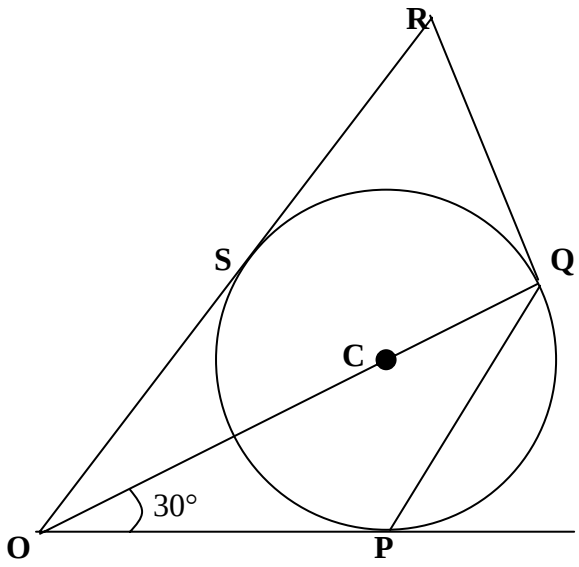
$2\angle ADC = \angle DPA + \angle DPB + \angle DAP + \angle DBP$

$2\angle ADC = 90^\circ + 90^\circ$

$\angle ADC = 90^\circ$

$\therefore AB \perp PC$

3. In the figure, C is the centre of the circle PQS. OR and OP are tangent to the circle at S and P respectively. OCQ is a straight line and $\angle QOP = 30°$.



(a) Show that $\angle PQO = 30°$. (3 marks)

(b) Suppose OPQR is a cyclic quadrilateral.

(i) Show that RQ is tangent to circle PQS at Q. (3 marks)

黎 Sir 提提你  :

(a) Construct a line CP

$\angle OPC = 90^\circ$ **(Tangent $\perp$ radius)**

$\angle POC = 180^\circ - 90^\circ - 30^\circ = 60^\circ$ **($\angle$ sum of Δ)**

$\angle PQO = \frac{1}{2}\angle PCO = 30^\circ$ **($\angle$ at centre twice $\angle$ at circumference)**

(b) $\angle SOC = 30^{\circ\circ}$ **(Tangent properties)**

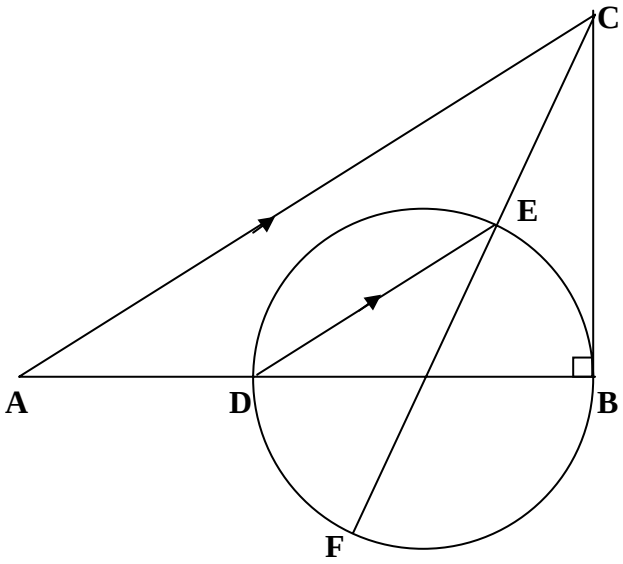
$\angle RQP + \angle SOP = 180^\circ$ **(oppo. $\angle$s cyclic quadrilateral)**

$\angle RQO + 30^\circ + 30^\circ + 30^\circ = 180^\circ$

$\angle RQO = 90^\circ$

$\therefore$ **RQ is tangent to circle PQS at Q.**

4. In the figure, ABC is a triangle right-angled at B. D is a point on AB. A circle is drawn with DB as a diameter. The line through D and parallel to AC cuts the circle at E. CE is produced to cut the circle at F.



(i) Prove that A, F, B and C are concyclic.

(ii) If M is the mid-point of AC, explain why $MB = MF$ **(6 marks)**

黎 Sir 提提你  **:**

a (i)

$\angle ACF = \angle DEF$ **(** $Corr\angle s, AC // DE$ **)**

$\angle DEF = \angle DBF$ **(** $\angle s$ **in same segment)**

$\therefore \angle ACF = \angle DBF$

Hence A, F B and C are concyclic. (Converse of $\angle s$ **in same segment)**

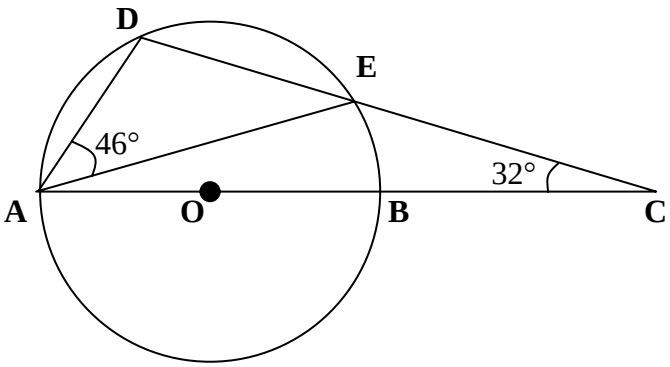
a (ii.) $\because$ **A, F, B and C are concyclic and** $\angle ABC = 90^\circ$

$\therefore$ **AC is a diameter of the circle AFBC.**

Hence M is the centre and MB, MF and radii of the circle AFBC

$\therefore$ **MB = MF.**

5. In the figure, a chord CD and a diameter AB are produced to meet at C.
If $\angle DAE = 46°$ and $\angle DCA = 32°$. O is the centre of the circle. Find $\angle DEA$.



(3 marks)

黎 Sir 提提你  :

Construct a straight line EB.

$\angle AEB = 90^\circ$ **(∠ in semi circle)**

$\angle EAC + 32^\circ = \angle DEA$ **(ext. ∠ of Δ)**

$\angle EAC = \angle DEA - 32^\circ$

$\angle DAE + \angle EAC + \angle DEA + \angle AEB = 180^\circ$ **(oppo. ∠ cyclic quadrilateral)**

$46^\circ + \angle DEA - 32^\circ + \angle DEA + 90^\circ = 180^\circ$

$2\angle DEA + 104^\circ = 180^\circ$

$2\angle DEA = 76^\circ$

$\angle DEA = 38^\circ$

The End.