

# Mathematics

# 數學科

## Quadratic Equations

## 二次方程



計數要小心，  
咪期望快一陣！

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二元聯立方程

課題	註釋 (整體課程)	註釋 (基礎部分)
5 一元方程。	方程與恆等式之區別。 一元線性方程。 利用因式法、公式法及圖解法解二次方程。根之性質。簡單應用題。 可變換為二次方程的方程。	方程與恆等式之區別。 一元線性方程。 利用因式法、公式法及圖解法解二次方程。根之性質。簡單應用題。
二元聯立方程。	觀察既定圖像的交點來解方程。解兩皆為一次方程，包括圖解法。以代數方法解一為一次，一為二次方程。	觀察既定圖像的交點來解方程。解兩皆為一次方程，包括圖解法。

<i>Syllabus Topics</i>	<i>Notes (Whole Syllabus)</i>	<i>Notes (Foundation Part)</i>
5 Equations in one unknown.	Distinction between identities and equations. Linear equation in one unknown. Solving quadratic equations by factorization, by formula and by graph. Nature of roots. Simple application problems. Equations which can be transformed to quadratic equations.	Distinction between identities and equations. Linear equation in one unknown. Solving quadratic equations by factorization, by formula and by graph. Nature of roots. Simple application problems.
Simultaneous equations in two unknowns.	Solving equations by reading intersecting points of given graphs. Solving two linear equations, including graphical method. Solving one linear and one quadratic equations by algebraic method.	Solving equations by reading intersecting points of given graphs. Solving two linear equations, including graphical method.

## (A) Distinction between identities and equations.

### 方程與恆等式之區別

#### i. Definition of Equation / Identity 方程/恆等式的定義

**Equation 方程:** an equality (等式) which is true for special value(s) of x (特別 x).

**Identity 恆等式:** an equality (等式) which is true for all values of x (所有 x).

#### ii. Useful Identities 有用恆等式:

a.  $a^2 + 2ab + b^2 \equiv (a + b)^2$

b.  $a^2 - 2ab + b^2 \equiv (a - b)^2$

c.  $a^2 - b^2 \equiv (a + b)(a - b)$

d.  $a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$

e.  $a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$

f.  $a^4 - b^4 \equiv (a^2 + b^2)(a + b)(a - b)$

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## Exam Type Questions:

### **a. Determine Identities & Equations 證明恆等式和方程**

e.g. Which is Equation? Which is Identity? 那些是方程? 那些是恆等式?

a.  $x^2 + 2x + 1 = 0$       b.  $x = 0$       c.  $x^2 + 2x + 1 = (x+1)^2$       d.  $a^2 - b^2 = (a+b)(a-b)$

### **b. Find Unknowns in identities 找出恆等式的未知數**

#### **Skill 1: Compare Coefficient 技巧 1: 比較係數**

e.g. Find k if  $(x-3)(x-1) \equiv x^2 - 4x + k$ .

Sol:

e.g. For what value(s) of x does the equality (等式)  $\frac{(x-1)(x+2)}{x-1} = x+2$  hold (成立)?

Sol:

e.g. Given the identity  $\frac{y+1}{(y-1)^2} + \frac{2}{1-y} - \frac{R}{(1-y)^2} \equiv \frac{2-Sy}{(1-y)^2}$ . Find the value of R and S.

Sol:

#### **Skill 2: Substitution 技巧 2: 代入法**

e.g. Find A, B and C if  $A(x+1)(x-2) + B(x-1) + C \equiv 3x^2 + 2x - 1$

Sol:

## (B) Linear equation in one unknown

### 一元線性方程

$ax + b = 0 \rightarrow$  Linear equation in one unknown 一元線性方程

e.g.  $3x + 7 = -6$

Sol:

e.g.  $\log x + \log 5 = \log 7$

Sol:

## (C) 3 ways to solve quadratic equations

### 解決二次方程的 3 個辦法

#### Exam Type Questions:

#### a. Solving Quadratic Equations 解決二次方程

#### Skill 1: By Factorization - Common Factor

#### 技巧 1: 因式分解 - 公因式

e.g. Solve 解  $(x-1)(x-4) = (x-4)$ .

Sol:

$$(x-1)\cancel{(x-4)} = \cancel{(x-4)}$$

$$(x-1) = 1$$

$$x = 2 \quad ??? \text{ What's wrong?}$$

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## Skill 2: By Factorization - Useful Identities

### 技巧 2: 因式分解 - 有用恆等式

e.g.  $4x^2 + 12x + 9 = 0$

Sol:

e.g.  $9x^2 - 16 = 0$

Sol:

## Skill 3: By Factorization - By Cross Method

### 技巧 3: 因式分解 - 十字法

e.g.  $\frac{6}{x+1} \left( \frac{1}{x+1} - \frac{1}{6} \right) = 2$

Sol:

e.g.  $x^2 - 3x + 2 = 0$

Sol:

## Skill 4: By Formula

### 技巧 4: 公式法

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } \Delta = b^2 - 4ac \rightarrow \text{Discriminant 判別式}$
---

e.g.  $4x^2 + 12x + 9 = 0$

Sol:

e.g.  $9x^2 - 16 = 0$

Sol:

## Skill 5: By Graph 技巧 5: 圖解法

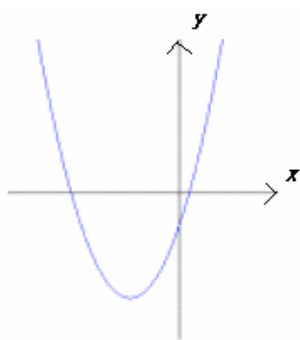
Two Steps: 兩個步驟:

1. Plot (繪畫) the graph of  $y = ax^2 + bx + c$
2. Find the x-intercept (x 截距) of  $y = ax^2 + bx + c$ , i.e. the roots of  $ax^2 + bx + c = 0$

\*\*\* Quadratic Equation 二次方程:  $ax^2 + bx + c = 0$

---> Special Case of Quadratic Function 二次函數的特別情況 !!!

i.e.  $y = ax^2 + bx + c$  when  $y = 0$



Quadratic Function:

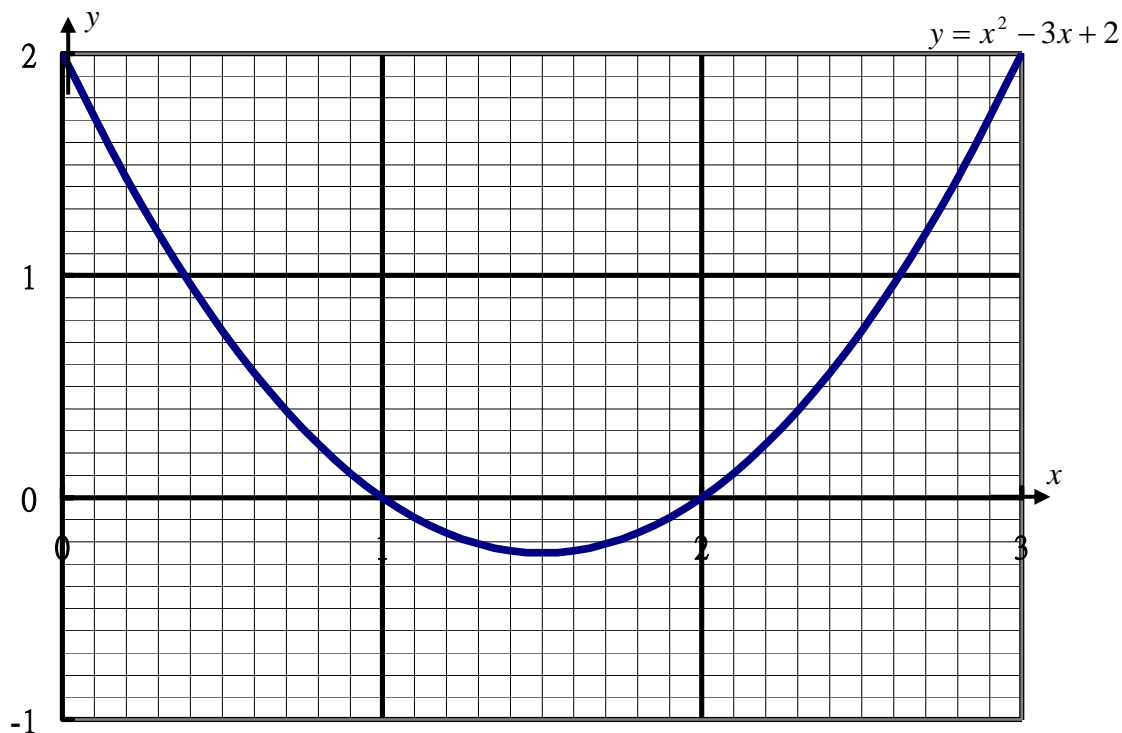
$$y = ax^2 + bx + c$$

Quadratic Equation:

$$ax^2 + bx + c = 0$$

e.g. Solve 解  $x^2 - 3x + 2 = 0$ .

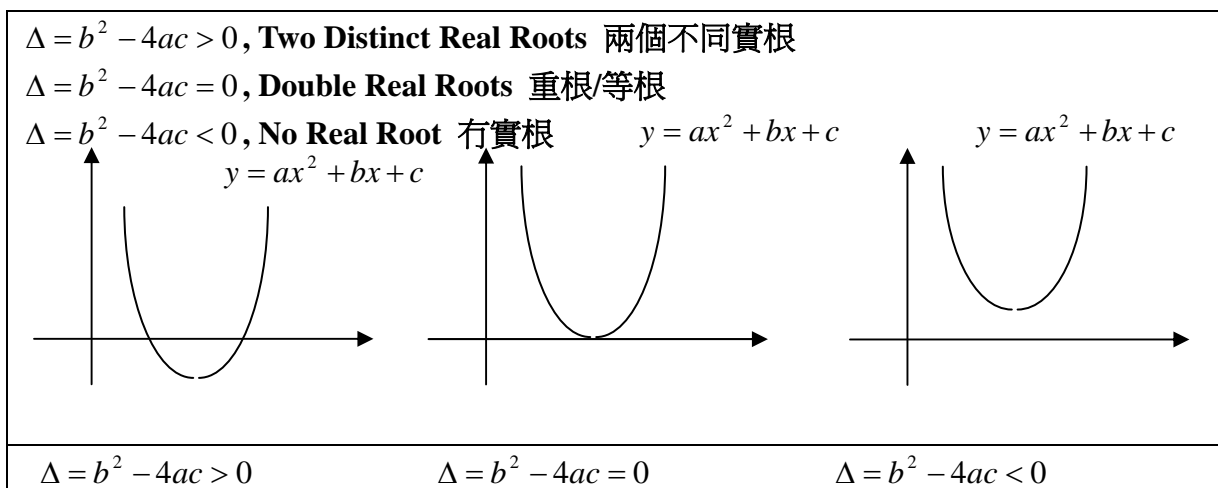
Step 1: Plot (繪畫) the graph  $y = x^2 - 3x + 2$



Step 2: From the graph, the roots of  $x^2 - 3x + 2 = 0$  are 1 and 2.

## b. Nature of Root 根之性質

### Skill 1: By Discriminant 技巧 1: 判別式



e.g. Find the range of  $p$  so that  $4x^2 + (p+3)x = 1$  has real roots.

求  $p$  值的範圍使方程  $4x^2 + (p+3)x = 1$  有實根.

Sol:

e.g. Find the value of  $k$  if  $3x^2 + 4x + k = 0$  has equal root.

求  $k$  值如果  $3x^2 + 4x + k = 0$  有等根.

Sol:

e.g. If  $9x^2 + kx + 4 = 0$  has equal positive roots (等正根), find  $k$ .

求  $k$  值如果  $9x^2 + kx + 4 = 0$  有等正根.

Sol:

## (D) Equations transformed to quadratic equations

可轉換為二次方程的方程

### Exam Type Questions:

#### **Skill 1: Fractional Equations** 技巧 1: 份數方程

e.g.  $\frac{1}{2+x} + \frac{2}{4x-1} = 1$

Sol:

e.g.  $\frac{6}{x-1} - \frac{8}{x+1} = 1$

Sol:

#### **Skill 2: Equations with degree larger than 2**

#### 技巧 2: 次方大過 2 的方程

e.g.  $x^4 + 2x^2 + 1 = 0$

Sol:

e.g.  $x^4 + x^2 - 20 = 0$

Sol:

#### **Skill 3: Equations with Surd form** 技巧 3: 根式方程

e.g.  $x - 2\sqrt{x} = 3$

Sol:

e.g.  $4x - 2 - 7\sqrt{x} = 0$

Sol:

## Skill 4: 技巧 4 Exponential Equations 指數方程

e.g.  $2^{2x} - 2^x - 6 = 0$

Sol:

e.g.  $12(4^{2x}) - 4(4^x) = 1$

Sol:

## Skill 5: 技巧 5 Logarithmic Equations 對數方程

e.g.  $(\log x)^2 + 2 \log x + 1 = 0$

Sol:

e.g.  $2 \log x - \log(3 - x) = \log 4$

Sol:

## (E) Applications of quadratic equations

### 二次方程的應用

#### Four Steps to deal with quadratic equations application problems

應付二次方程應用題的四個步驟

1. Let the unknown be  $x$  設未知數是  $x$
2. Set up a quadratic equation in  $x$  設立未知數是  $x$  的二次方程
3. Solve the quadratic equation 解決二次方程
4. Answer the question 回答問題

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## **Exam Type Questions:**

### **a. Applications of quadratic equations 二次方程應用題**

#### **Skill 1: Compound Interests 技巧 1: 複利息**

e.g. Andy deposits (儲存) \$10000 into the saving deposit account (儲蓄戶口) on his 28<sup>th</sup> birthday and \$9000 half year later. On his 29<sup>th</sup> birthday, there is \$22000 in his account. The annual interest rate (年利率) for his saving deposit account is  $x\%$  compounded half-yearly (半年結算一次). Find the annual interest rate  $x\%$ .

(Andy 28 歲生日時在儲蓄戶口儲存了\$10000, 半年後再存入\$9000. 在 29 歲生日時, 儲蓄戶口累積了\$22000. 假設半年結算一次, 年利率為  $x\%$ . 求年利率  $x\%$ .)

Sol:

#### **Skill 2: Length of Rectangle 技巧 2: 長方形長度**

e.g. Consider a rectangular farm is  $864 \text{ m}^2$ . If the length of a farm is increased by 4 m and the width of a farm is reduced (減少) by 3 m, the area (面積) of the new farm is remained the same (保持不變). Find the original dimension (原來尺寸) of the farm.

(考慮一長方形農場面積為  $864 \text{ m}^2$ . 如果農場長度為增加了 4 m 和闊度減少了 3 m, 農場面積保持不變. 求農場的原來尺寸.)

Sol:

### Skill 3: Application of Pythagoras' Theorem

#### 技巧 3: 畢氏定理的應用

e.g. A ship leaves port A, sails east to port B, and then north to port C, with a total distance (總距離) of 119 km. Tomorrow, the ship sails directly (直接地) from port C back to port A with a total distance of 91 km. Find the distance between port A and B. (一艘輪船駛出港口 A, 向東方駛向港口 B, 再向北駛向港口 C, 總距離為 119 km. 第二天, 那艘輪船直接地由港口 C 返回港口 A 而總距離為 91 km. 求港口 A 和港口 B 之間的距離.)

Sol:

### Skill 4: Speed Problem 技巧 4: 速度問題

e.g. Tom and Jimmy is driving their car from his school to Mary's home, the distance (距離) is 112 km. Jimmy start to drive his car after  $3\frac{1}{3}$  hours than Tom, but his speed (速度) is 10 km/h faster than Tom's speed. If they arrive at Mary's home at the same time, find the speed of Tom and Jimmy.

(Tom 和 Jimmy 由學校一起駕車到 Mary 家, 距離為 112 km. Jimmy 比 Tom 遲  $3\frac{1}{3}$  小時出發, 但他的速度比 Tom 的速度快 10 km/h. 如果他們同時到達 Mary 家中, 求 Tom 和 Jimmy 的速度.)

Sol:

## **(F) Simultaneous equations in 2 unknowns**

### 二元聯立方程

$$\begin{cases} y = ax^2 + bx + c \\ y = mx + c \end{cases} \quad \text{or} \quad \begin{cases} y = m_1x + c_1 \\ y = m_2x + c_2 \end{cases}$$

### **Exam Type Questions:**

#### **a. Solving Simultaneous Equations 解聯立方程**

##### **Skill 1: Substitution 技巧 1: 代入法**

e.g.

Sol:

##### **Skill 2: Elimination 技巧 2: 消元法**

e.g.

Sol:

### Skill 3: Graphical Method 技巧 3: 圖解法

Intersection point of the graph = Solutions of simultaneous equations

圖上的相交點 = 聯立方程的解

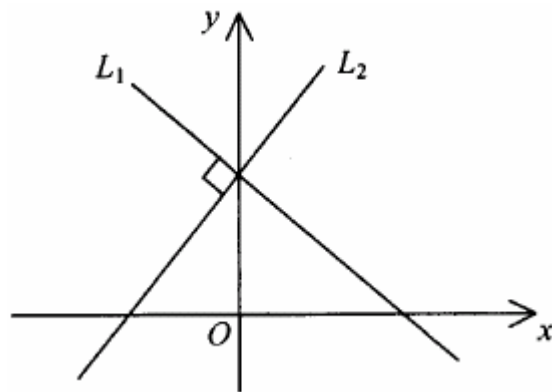
e.g.

$L_1$  and  $L_2$  are 2 straight lines (直線)

intersecting (相交) at a point on the y-axis (y 軸). If the equation (方程) of  $L_2$  is

$x - 2y - 1 = 0$ , find the equation (方程) of  $L_1$

Sol:



### b. Application Problems 應用題

#### Skill 1: Price/Quantity Calculation of 2 Goods

##### 技巧 1: 計算兩種貨物的價格/數量

e.g. The prices (價格) of a rubber and a ruler are \$2 and \$3 respectively (各自地). If Andy use totally (總共) \$46 to buy some rubbers and rulers for his students. The total number of rubber and rulers is 20. Find the number of rubbers and rulers.

(擦子膠和間尺的價錢是\$2 和\$3. 如果 Andy 用了\$46 去購買一些擦子膠和間尺, 總數為 20 件. 求擦子膠和間尺的數量.)

Sol:

## Skill 2: Length/Area of Rectangle

### 技巧 2: 長方形長度/面積

e.g. The area (面積) of the rectangle is  $60 \text{ cm}^2$ . The width (闊度) and the length (長度) of a rectangle are  $x \text{ cm}$  and  $y \text{ cm}$  respectively (各自地). The length is  $1 \text{ cm}$  longer than (長過) half of its width (闊度的一半). Find the length and width of the rectangle.  
(一長方形面積為  $60 \text{ cm}^2$ . 假設闊度是  $x \text{ cm}$ , 長度是  $y \text{ cm}$ . 長度比闊度的一半長  $1 \text{ cm}$ . 求長度和闊度.)

Sol:

## Skill 3: Double-digit number 技巧 3: 雙位數字

e.g. A two-digit (雙位) student number is increased (增加了) by  $18$  when its digits are reversed (數位對調). If the sum of the squares of the digits (數位次方之和) is  $130$ , find the original student number (原來學生數目).  
(當一個雙位學生數字的數位對調後, 數值增加了  $18$ . 如果數位次方之和是  $130$ , 求原來學生數目.)

Sol: